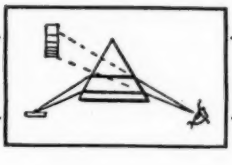
Journal of the OPTICAL SOCIETY *of* AMERICA

VOLUME 30
No. 2



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Published monthly
at Prince and Lemon Sts., Lancaster, Pa.
by the
AMERICAN INSTITUTE OF PHYSICS
Incorporated
175 Fifth Avenue, New York, N. Y.
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THE OPTICAL SOCIETY OF AMERICA was organized in 1916 "*to increase and diffuse the knowledge of optics in all its branches, pure and applied, to promote the mutual interests of investigators of optical problems, of designers, manufacturers and users of optical instruments and apparatus of all kinds and to encourage cooperation among them.*" It aims to serve the interests of all who are engaged in any branch of optics from fundamental research to the manufacture of optical goods.

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For further information concerning the Optical Society of America address the Secretary, Professor Arthur C. Hardy, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Journal of the
OPTICAL SOCIETY *of* AMERICA

VOLUME 30 FEBRUARY, 1940 NUMBER 2

Editorial

THE *Journal of the Optical Society of America*, like the Optical Society itself and at least a dozen other American institutions, is realizing ever more fully its loss in the passing of Floyd K. Richtmyer. The deprivation felt by those who were fortunate enough to have contact with him as a scientist, as an administrator, as an editor, as a teacher, as an author, or just as a person, will gradually be assuaged by time. But some things must be irrevocably lost because he is no longer with us to accomplish them. For this loss compensation can come only through increased striving toward effective effort on the part of those who remain.

There are many ways in which a man can further the advancement of science. He can labor assiduously in laboratory or at desk; he can direct and coordinate the efforts of a staff of workers; he can assist others in learning how science has progressed; he can serve on committees and boards. Professor Richtmyer did all these things and did them well, but he did more. He had that extra gift, vouchsafed to only a few, of being able to further the advancement of science merely by his presence.

Professor Richtmyer was not an exceptionally large man, yet many of his friends now find themselves realizing that they had always subconsciously considered him of almost gigantic stature. Some of this impression doubtless came from the ranginess of his form and features, yet more of it arose from the psychological aura of sturdy dependability and imperturbability which surrounded him; one felt that no matter what shifting of sands might come from wind or water, here would remain a sturdy rock.

A very human person, Professor Richtmyer had that common attribute of the scientist of being embarrassed at the necessity of providing "small talk," an embarrassment which he spontaneously covered by a friendly and disarming chuckle which will long be cherished in the memories of his friends as one of his most lovable characteristics. Yet when the responsibility of officiating at a meeting or a banquet was his, no one could rise to more adequate heights, or set more felicitously the tone of an occasion.

So numerous and diverse were his talents that Professor Richtmyer found the demands on his time ever increasing, yet he was never too busy to give of himself whenever the demand should come. Many a young scientist, finding himself enmeshed in a problem which refused to be untangled, has been stimulated and reheartened by a word or two from this man who so strongly radiated forceful resourcefulness.

Such was the individual who directed the destinies of this Journal during the past seven years. His obituary has appeared here and in other journals—these further words represent merely the natural reflections of a person finding himself suddenly charged with even a small part of the responsibilities of so talented a guide. The directors of the Optical Society must have realized that there is no possibility of finding anyone who can fill Professor Richtmyer's position as Editor, yet the Journal must continue and will continue.

We can only hope that this loss will make all who have concern for the advancement of optics more fully conscious of their common aim. From this should spring an increasing sense of responsibility to the organizations and publications in which this aim is embodied.

Comparative Anatomical Studies of the Eye with Especial Reference to the Photoreceptors*

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(Received November 20, 1939)

IN 1866 Max Schultze[1] discovered that the vertebrate eye, and particularly that of man, possesses two types of visual elements, namely, rods and cones, and that these elements subserve different visual functions. Subsequent elaborations of these findings were made by Parinaud and von Kreis and led, respectively, to the so-called theory of the double retina of Parinaud, and the duplicity theory of von Kreis. The essential features of these findings are that the rod elements are concerned with colorless vision at low intensities of light and that the cones constitute the apparatus concerned with vision at high intensity, as well as with the perception of color. The cones are not assumed to be useless at night but relatively so for the majority of them possess thresholds which are much higher than the highest thresholds of the rods. This theory, since its original conception has been generally regarded as well substantiated, notably from many physiological measurements of the major function of vision such as dark adaptation, intensity discrimination, visual acuity and the phenomenon of flicker. It is also borne out by the facts of diurnal and nocturnal vision. In fact so closely correlated is the mode of life of the animal with the structural make-up of the retina, that I think from a histological examination of the retina one can predict with reasonable assurance something of the habits of the animal as well as its visual ability.

Just what constitutes a rod or a cone would seem to be a relatively simple matter, but the great variety in the form of these cells in different animals has been an obstacle to a general classification. In fact some relatively recent observations by Mademoiselle Verrier[2] are so interpreted as to regard the so-called rods and cones as representing extreme structural variations of a single photoreceptor cell—thus doing away with the concept of morphological and physiological duality in the retina.

The generally described regional layers of the retina and the structural elements composing them are shown in Fig. 1. It is generally regarded as fact that each cone makes a centripetal synaptic connection with a single bipolar cell, whereas several or many rods become related to the dendritic endings of a single bipolar cell (Fig. 2). Such a condition presents favorable structural situations providing for summated conduction for the rods and individual isolated conduction for the cones, thus not only enhancing rod vision at low intensities, but favoring greater visual acuity of the cones at high intensities. This condition, however, is not borne out by many published illustrations.

THE STRUCTURE OF VISUAL CELLS

Typically the cone is a conical-shaped element possessing two major portions—an inner and an outer segment. It is the conical shape of the outer segment which has long been used to distinguish this cell as a cone, whereas the outer segment of the rod is typically cylindrical.

The cone inner segment typically possesses an ellipsoid and a myoid element. Frequently, in addition, there is present a highly refractive globular structure called the paraboloid. A spherical oil drop is present in many forms and is situated at the distal end of the inner segment (Fig. 3). In the rod there is frequently present a refractive disk occupying a position corresponding to the oil drop of the cone. Many forms possess an ellipsoid, a paraboloid may or may not be present (Fig. 4). Both rods and cones possess a myoid element which extends through the external limiting membrane.

Although structural differentiation is usually based upon the form of the outer segment (cone-

* Invited paper presented at the twenty-fourth annual meeting of the Optical Society of America held at the Lake Placid Club, Essex County, New York, October 13, 1939.

[1] M. Schultze, *Zur Anatomie und Physiologie der Retina* (Arch. mikr. Anat., 1866), Bd. 2, S. 175.

[2] M. L. Verrier, "Recherches sur l'histophysiologie de la rétina des vertébrés et les problèms qu'elle soulève," Bull. Biol. de France et de Belgique. Supp. 20, 1 (1935).

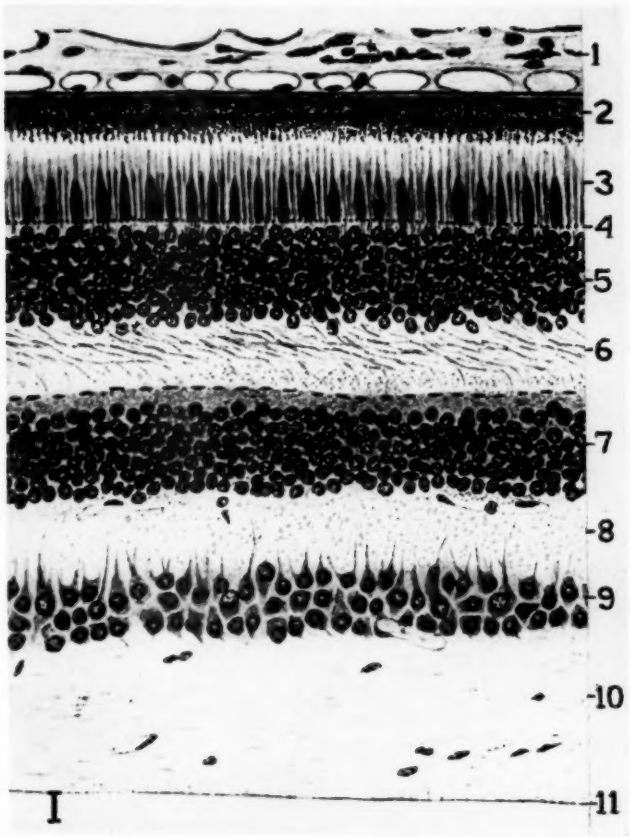

FIG. 1. Section of the human retina (from Fig. 475, Bailey's *Textbook of Histology*, 9th Edition). 1, Choroid; 2, pigment epithelium; 3, rods and cones; 4, external limiting membrane; 5, external nuclear layer; 6, external molecular (plexiform) layer; 7, internal nuclear layer; 8, internal molecular (plexiform) layer; 9, ganglionic cell layer; 10, nerve fiber layer; 11, internal limiting membrane.

shape *vs.* rod-shape) it has been claimed by some that it is the mode of centripetal connection with the bipolar cell which serves as the real basis for classification. Upon this basis all cones are supposed to possess dendritic endings and the rods knob-like or molecular endings regardless of external form (Fig. 5). This is the situation usually illustrated in textbooks for the human retina. Diurnal birds, however, possess visual cells which, from their general form, should be called rods, and are called rods by Franz,[3] yet they possess terminal dendritic connections typical of cones (Fig. 5). On the same basis Pütter[4] claims that all reptiles possess cones only, yet certain groups of reptiles are strictly nocturnal in their habits (e.g. certain lizards and snakes) and their visual cells have all the other structural characteristics of rods. Further, it is common knowledge to histologists that the cones at the fovea centralis of man and many animals are attenuated, cylindrical-shaped elements and certainly look more like rods than cones; yet regardless of their shape or centripetal connections, their known function fits in with the physiology of cone vision. In many groups of vertebrates (fishes, amphibia, birds) which possess both types of visual cells, a definite distinction can be made between the two types, regardless of variations in shape, on the basis of their directly opposite (inverse) photomechanical or phototropic responses to light. The cone typically shortens in the light and elongates in the dark. The rod exhibits responses in exactly the reverse order to the same conditions of illumination.

[3] V. Franz, 1913 Sehorgan. (In Oppel's Lehrb. d. vergl. mikros. Anat. d. Wirbeltiere. G. Fisher).

[4] A. v. Pütter, *Organologie des Auges* (Wm. Englemann, 1908).

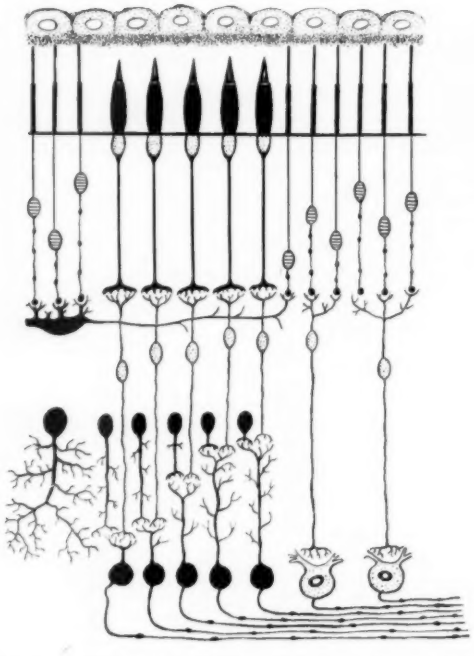

FIG. 2. Plan of the retinal neurones as shown by the Golgi method, showing individual connection of the cones with a single bipolar cell, and multiple connections of the rods with a single bipolar cell (redrawn from Greef and Cajal).

This phototropic reaction, however, apparently does not take place in mammals and man.

RETINAL STRUCTURE AND ANIMAL HABITS

The eyes among the great majority of vertebrate animals possess both rods and cones, yet certain groups exhibit only one type of visual cell (either rods or cones) and it is most interesting to note that the habits of animals are correlated with retinal structure. Pure cone retinae such as exists in the majority of reptilian forms are indicative of diurnal mode of life and forms possessing this type of retina appear to have little capacity for vision in very low intensities of light. On the other hand, such reptilian forms as the Gecko, which are cone-free, the crocodiles, (with their rod-rich retina and assisted by the presence of a light reflecting tapetum) and many other forms characterized by the complete or relative absence of cones, possess a mechanism particularly adapted for nocturnal life. Certainly we are all aware from personal observations that our domestic neighbors, the rats and mice, are much more active at night than in daytime, and the bats which possess a pure rod retina begin their activity with approaching dusk as do also the Geckos. Upon this same basis of structural difference in the retina, birds also become divisible into two functional groups, *viz.*, day birds and night birds. It is a well-known fact that certain birds migrate only at night, and almost yearly we hear of scores of migrating night birds which encounter death on the wing, by flying into strongly illuminated tall buildings, apparently dazzled by the bright light. Examples of mixed retinae, and of those possessing a single type of photoreceptor cell are given in Figs. 6–11.

Whereas mammals in general possess both rods and cones, it is of interest to know that among the Lemuroids and Tarseoids which are strictly nocturnal in their habits, their retinae are devoid of cones. These creatures which belong to the lower branch of the anthropoid stem leap from bough to bough at night only, when they also do their feeding. Recently I have been fortunate to come into possession of a living Galago and the slow moving Loris both of which belong to the Lemur family. I found it difficult to arouse either of these forms during the daytime. Food was consumed only at night. Photographs of the slow moving Loris were difficult to take, for when light sufficiently strong for photographs was flashed on, the creature buried his face between his front legs and a certain amount of physical irritation or disturbance was necessary before he would be aroused and open his eyes for another trial. Both of these strongly nocturnal animals possess a pure rod retina with no evidence of any cones (Figs. 12, 13). Furthermore, they are entirely devoid of a fovea centralis which would be expected upon the basis of their nocturnality. Woollard[5] who described the retina of Tarsius, describes the presence of an area centralis (a local retinal thickening) and regards it as the forerunner of a true fovea, but in his observations he was apparently dealing with artificial foldings in the retina due to shrinkage. We are apparently

[5] H. H. Woollard, "The Anatomy of Tarsius Spectrum," Proc. Zool. Soc. London, p. 1071 (1925).

safe in saying that these strongly nocturnal prosimian forms possess rods only, and totally lack an area centralis and a fovea, which are found only in those diurnal forms which possess an abundance of cones and where the cones are concentrated into these localized regions for vision at high intensities and for greater visual acuity.

The eyes of these prosimian forms, in addition to their characteristic lack of cones, present interesting features as compared with those of monkeys and man. Typical of nocturnal animals, the cornea presents a marked curvature. In Galago and Nyctecebus, both of which are

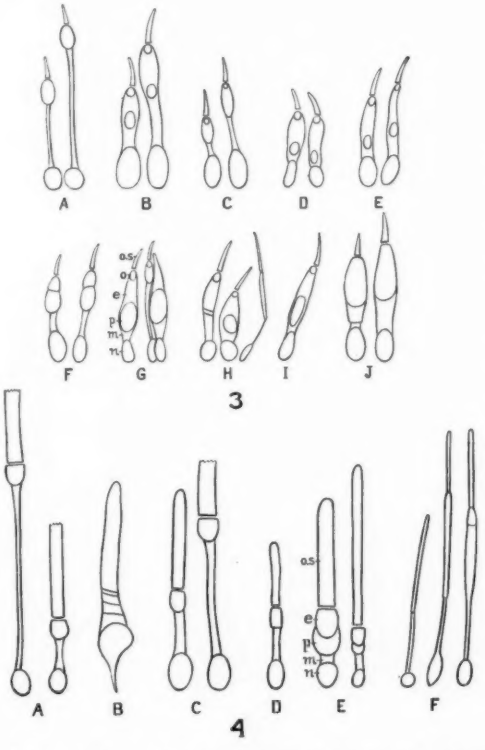

FIG. 3. Cones from various vertebrates. *A*, Teleost (Ameiurus); *B*, Urodele (Amblystoma); *C*, Anuran (Rana); *D*, Chelonia (Chrysemys); *E*, Lacertilia (Phrynosoma); *I*, Aves (Gallus); J, Mammalia (Sus and Homo)—*n*, nucleus; *m*, myoid; *p*, paraboloid; *e*, ellipsoid; *o*, oil drop; *os*, outer segment.

FIG. 4. Rods from various vertebrates. *A*, Teleost (Ameiurus); *B*, Urodele (Amblystoma); *C*, Anuran (Rana); *D*, Crocodilia (Alligator); *E*, Lacertilia (Gecko); *F*, Mammalia (Mus, Sus, Homo)—*n*, nucleus; *m*, myoid; *p*, paraboloid; *e*, ellipsoid; *os*, outer segment.

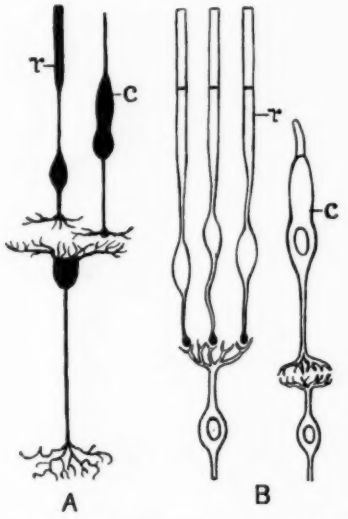

FIG. 5. *A*, Dendritic centripetal endings of both rods (*r*) and cones (*c*) in birds, according to Franz (1913); *B*, molecular centripetal endings of rods (*r*), and dendritic endings of cones (*c*) according to Greef (v. Pütter, 1908).

Lemurs, and in Tarsius, it occupies about one-third of the globe whereas in the human it is between one-fifth and one-sixth. The lens in Galago and Nyctecebus in relation to the size of the whole eye is enormous. The size relation of lens to vitreous in Galago is as 1 : 1.63 whereas in the marmoset and in man it is as 1 : 9. Accompanying this enormous lens, the ciliary muscle is notably large. This muscle, however, in the marmoset, is decidedly larger in relation to lens size than in the other forms studied, which would point to a superior accommodating power for this form. However, when one compares the relative size of the ciliary muscle with the whole eye one sees that it is relatively largest in Galago—it being four times as large as in the rhesus monkey and man, and nearly twice as large as this muscle in the marmoset. The size relations of the various eye structures are shown in Fig. 14 (*a–e*) and the ratios are expressed in Table I.

VISUAL ACUITY

Insofar as the retina is concerned, the degree of visual acuity which an animal may enjoy or the capacity for detailed vision is dependent, apparently, upon its structure. The fineness of detail which a photographic plate can register

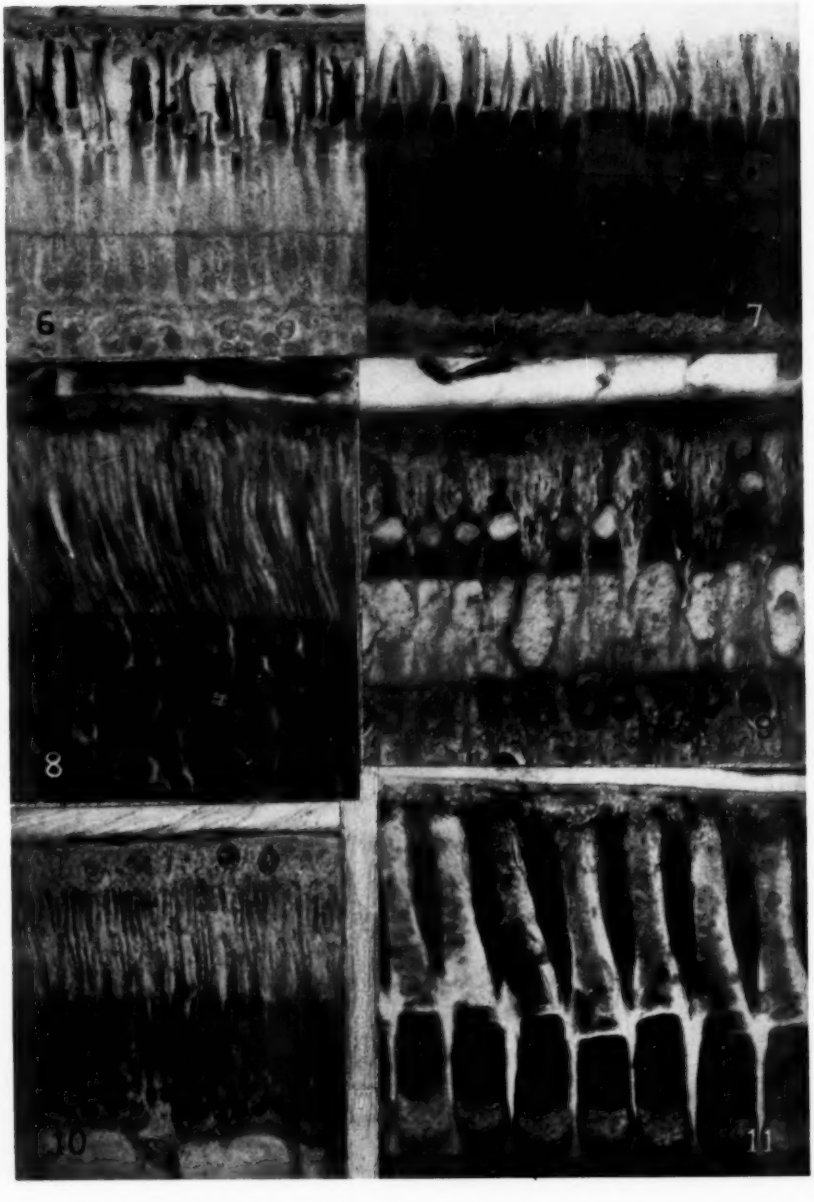

FIG. 6. Photomicrograph showing rod and cone visual cells of the chicken. ×630.
FIG. 7. Photomicrograph of the retina of the rhesus monkey, showing rods and cones. ×500.
FIG. 8. Photomicrograph of 8-day old rat retina, showing scanty epithelial pigment. ×850.
FIG. 9. Photomicrograph showing pure cone retina of the turtle. Note large ellipsoids and oil droplets (*Vide* Fig. 3). ×850.
FIG. 10. Photomicrograph of the dogfish (Elasmobranch) retina, showing rods only and absence of epithelial pigment. ×430. From preparation by R. L. Carpenter.
FIG. 11. Photomicrograph of the pure rod retina of the Gecko. Note heavy epithelial pigment between the outer segments of the photoreceptors. ×850.

depends upon the structure of the emulsion. If the particles are fine and close together, one obtains much greater detail than when the receiving elements are large and far apart. The same situation obtains in the retina. A low visual acuity signifies that the average distance between the photoreceptors is large, whereas a high visual acuity signifies that the distance is small. From this standpoint the vertebrate retina exhibits striking variations. Since the number of photoreceptors is fixed structurally in any given region of the eye, it is clear that the number of elements per unit area must vary functionally in order to mediate the great variation in visual acuity known to accompany changes in illumination. This has been shown by a number of investigators.

Regardless of whether one is dealing with a double retina or with one made up of one kind of visual cell, it is typical or universal that the photoreceptors are more numerous per unit area in the fundus of the eye—particularly in the visual axis—than toward the ora serrata. In some of the pure cone retinae such as in the turtles there is a small region in the optical axis of the eye where the cones are definitely smaller and more numerous. The increased numbers of cone nuclei, therefore, cause a bulging and thickening of the external nuclear layer. This region is called the area centralis retinae and appears to be the forerunner of the fovea centralis (Fig. 15). The concentration of numerous finer receiving elements in this area no doubt presents a structural condition enhancing greater visual acuity than in those eyes when it is not present.

In the diurnal lizards a true fovea centralis has developed and when one compares the highly developed fovea of such animals as the horned toad and the chameleon with that of man, one can only be led to believe that in the former the conditions for high visual acuity are more nearly met than in human beings. The cones in the chameleon fovea are extremely long, slender and closely packed. So deep is the foveal depression that all of the nuclear layers are displaced laterally (Fig. 16). In the monkey and human fovea, the depression is not nearly so deep, the external nuclear layer is continuous over the foveal pit and the cones are not so attenuated nor so closely packed as in the chameleon (Figs. 17, 18). From this it would appear that the human fovea is inferior to that of the chameleon. Many birds possess a fovea centralis, particularly the rapacious birds (Fig. 19). A less developed temporal fovea is also frequently present. The swallow likewise has a double fovea—one central and one temporal (Fig. 20). The marked visual acuity of these birds is well known. According to Walls the hawk fovea possesses one million cones per sq. mm. This would necessitate a solid aggregation of cones 1μ in diameter.

Measurements of the human fovea show that the average distance between the centers of two adjacent cones is 2.5μ which would mean approximately 160,000 elements per sq. mm. How much greater the resolving power is in a unit area with

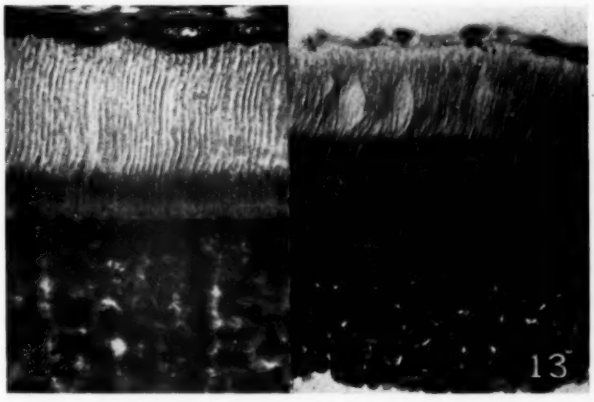

FIG. 12 (left). Photomicrograph of the pure rod retina of the nocturnal lemur (Galago mala). ×630.

FIG. 13 (right). Photomicrograph of the pure rod retina of the nocturnal lemur (Nyctecebus tardigrada). ×450. Note absence of epithelial pigment.

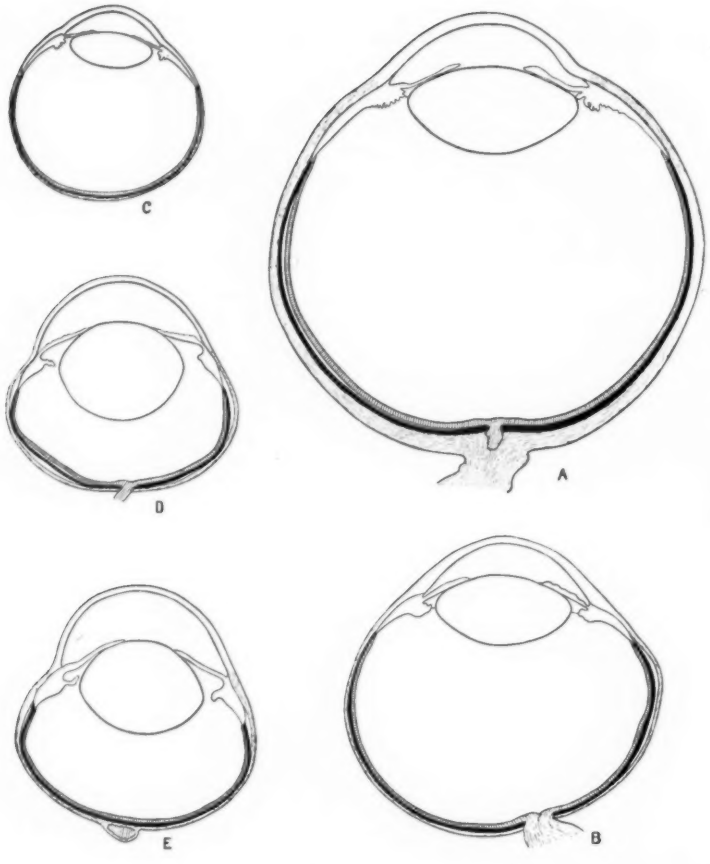

FIG. 14. *a*. Diagrammatic meridional section of the human eye. ×3. *b*. Diagrammatic meridional section of the rhesus monkey eye. ×3. *c*. Diagrammatic meridional section of the marmoset eye. ×3. *d*. Diagrammatic meridional section of the eye of the nocturnal lemur (Galago mala). ×3. *e*. Diagrammatic section of the eye of the nocturnal lemur (Nyctecebus tardigrada). ×3.

the elements 1μ apart as compared with 2.5μ, I am not in a position to say. A physicist friend informs me that the diffraction pattern sets a limit on resolution and this depends upon such factors as size of aperture and distance from lens to receiving surface. Thus it appears from this point of view that the fovea of reptiles and birds represents a more highly developed structure than in monkeys and man.

In a recent paper, Walls[6] has advanced some stimulating ideas regarding the significance of the foveal depression. It has been generally regarded that the attenuation or displacement of the retinal layers at the fovea presents a condition whereby light may reach the foveal cones unimpeded, i.e., without passing through the various retinal layers. He brings up the very pertinent point that in an afoveate area centralis, such as in the turtle, the retina at this point is thicker than elsewhere yet in spite of this, the

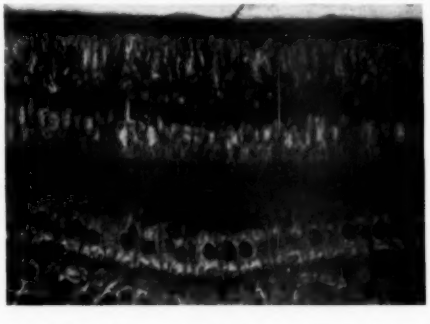

FIG. 15. Photomicrograph showing area centralis retina of the turtle. ×430.

[6] G. Walls, "Significance of the Foveal Depression," Arch. Ophthal. **18**, 912 (1937).

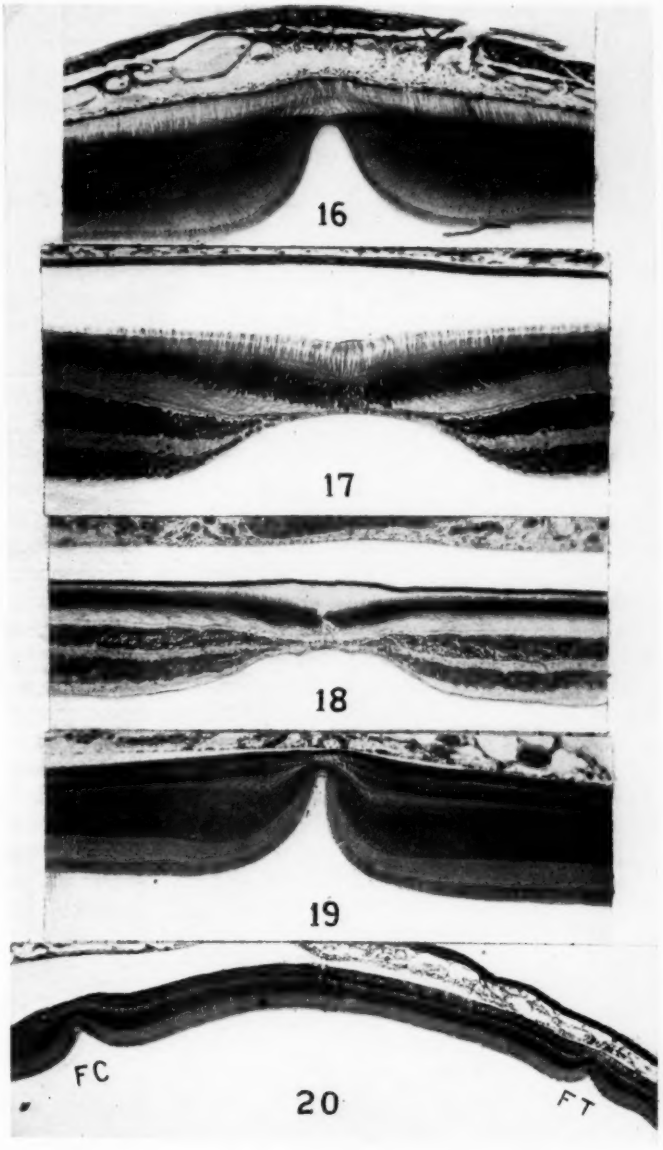

FIG. 16. Photomicrograph of the fovea centralis of the pure cone retina of the chameleon. ×50.

FIG. 17. Photomicrograph of the fovea centralis of the marmoset (monkey). ×100.

FIG. 18. Photomicrograph of the fovea centralis of the human retina. ×50.

FIG. 19. Photomicrograph of the fovea centralis of the hawk. ×50.

FIG. 20. Photomicrograph showing fovea centralis (FC) and fovea temporalis (FT) of the swallow. From collection of R. L. Carpenter.

resolving power of the area centralis retinae is greater. This would suggest that the production of a thin spot *per se* is not really so important. If the displacement of retinal layers in order to admit light were of paramount importance then the fovea of monkeys and man should be superior to that of reptiles and birds for in the latter, whereas the fovea is deeper, it is not so broad as in the former groups. Assuming that the retina has a higher index of refraction than the vitreous, Walls believes that the limiting membrane of the foveal clivus constitutes a refractive surface. The highly convexiclival surface refracts the light, thus broadening the retinal image and bringing into play a larger number of visual cells (Fig. 21). By reason of this optical property

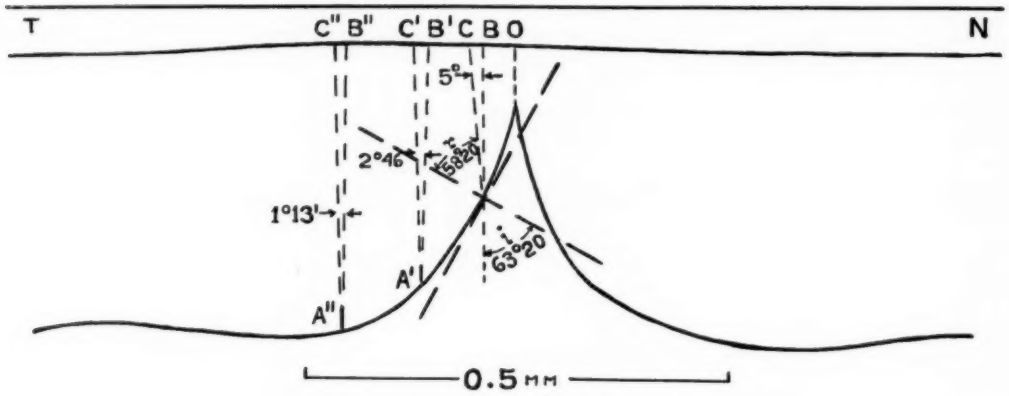

FIG. 21. Convexiclivate fovea of Saurians to illustrate refraction of light along the foveal clivus and spreading of the image on the foveal photoreceptors, according to Walls (1937). Figure redrawn from Walls.

there then occurs at the fovea a mechanism which increases the resolving power over that produced by the afoveate area centralis. From this point of view the convexiclivate fovea of lizards and birds represents a mechanism superior to the concaviclivate fovea of monkeys and man. It remains, I presume, to see whether this concept will stand the test of those who examine it from a purely optical viewpoint. The theory, however, seems very logical.

Time does not permit the discussion of many other interesting morphological features of the retina. It is of importance to note, as we view the various groups of vertebrates, that there exists a great diversity of structure, particularly as regards the presence or absence of rods and cones, as well as the occurrence and characteristics of the fovea centralis. These and many other anatomical features become matters of first importance in a comparative study of retinal adaptation, and also give a structural foundation which appears to be a prerequisite to an adequate study of the light perceiving value of the retina.

Although the eyes of vertebrates are constructed upon a common generalized plan, nature has deviated considerably from the common architectural scheme, not only with respect to the photoreceptor mechanism but, as well, with the light-collecting and focusing mechanisms. Thus nature's experiments stand before us to study and to evaluate in terms of our knowledge and applicability of physicochemical laws.

Spectral Luminosity Factors*

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(Received November 23, 1939)

As a result of extensive measurements by the flicker and equality-of-brightness methods, mostly in America, a certain set of average luminosity data was adopted by the International Commission on Illumination in 1924. These I.C.I. luminosity factors were incorporated in the 1931 I.C.I. standard observer and coordinate system for colorimetry and were accepted by the International Committee on Weights and Measures in 1933. In the period from 1935 to 1938, however, the validity of these factors was challenged by several German investigators, who believed as a result of their measurements that a new set of standard values should be adopted based on results by the flicker photometer with pure cone vision (1.2° field and daylight adaptation). The German measurements, however, were not wholly consistent and were in conflict with similar recent measurements in England and America. At the 1939 meetings of the I.C.I., the Germans withdrew their opposition to the 1924 I.C.I. luminosity factors and the Commission formally reaffirmed their validity in the photometry of lights of different colors.

I. INTRODUCTION

THE subject of spectral luminosity factors has been given considerable attention in Germany during the past few years. The subject has also been given active consideration by the Hungarian secretariat committee on visual photometry of the International Commission on Illumination; and, at the meetings in Holland last summer, the committee presented an excellent résumé of the recent work and the Commission took important formal action in this connection. It is hoped that the present paper will serve to inform those interested regarding the status of this important phase of photometry and colorimetry.

The term "luminosity" is used in the present paper in accordance with the following definitions.

"It was agreed that the word, *luminosity*, should supersede the word, *visibility*, for expressing the relation between luminous flux and radiant flux. The *luminosity* of a sample of radiant flux is, therefore, defined as the luminous efficiency of that radiant flux and is expressed in lumens per watt. If expressed as a function of wave-length, it is designated as *spectral luminosity*. If expressed relative to that at the wave-length of maximum luminosity, it is designated as *relative spectral luminosity*. These relative values may be designated as the *relative luminosity factors*, the *luminosity curve*, or in other abbreviated form where the context makes the meaning unambiguous. In accord with this definition, the use of luminosity for the product of spectral radiant flux by spectral luminous efficiency, or for similar products, should be discontinued."[42]

This use of the term, luminosity, has been followed by Dr. W. E. Forsythe in the book, *Measurement of Radiant Energy*,[50] and by Dr. L. A. Jones,[51] Chairman, Optical Society of America Committee on Colorimetry, who states: "For many years there has been considerable dissatisfaction with the use of the word 'visibility' as at present defined. Its present meaning is an extremely restricted one and the term is used extensively in reference to other aspects of visual experience. There has been a marked increase in the sentiment both in this country and abroad to substitute for the word 'visibility' as at present defined . . . , the word 'luminosity'. This committee feels that such a step is desirable. We therefore propose that the term *luminosity* be defined as the luminous efficiency of radiant energy. This action is in accord with the action reported by Dr. W. E. Forsythe,"—to which reference has already been made. Symbols K_λ and V_λ are both in common use for this function.

No attempt is made herein to cover the broad field of heterochromatic photometry, of which the measurement of spectral luminosity may be con-

* Invited paper presented at the twenty-fourth annual meeting of the Optical Society of America held at the Lake Placid Club, Essex County, New York, October 13, 1939.

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sidered a part. But, while only a part of this broader subject, the determination of the relative spectral luminosity factors is of fundamental importance both in heterochromatic photometry and in colorimetry, and without the establishment of accepted factors both the measurement and the specification of chromatic lights and chromatic objects would be seriously handicapped.

It is well known that the spectral luminosity curves of different individuals may differ considerably. This is illustrated in the composite plot of the luminosity curves of the 125 observers used by Coblentz and Emerson.[12] It is furthermore well known that the luminosity curve for any individual shifts considerably in the spectrum with varying experimental conditions, such as brightness and size of photometric field, these shifts being commonly designated as the Purkinje or inverse Purkinje effects. Time will not permit adequate consideration of this broad field, and the paper is accordingly restricted largely to a consideration of those determinations which have entered into the establishment of the standard luminosity factors for daylight (photopic) vision, and of the more recent determinations which support or disagree with the established factors. Those who may wish to investigate the Purkinje and related phenomena at both high and low brightnesses, studying the luminosity curve from the theoretical rather than the statistical point of view, will find a voluminous literature on the subject.*

II. Methods and Technique

The principal methods of heterochromatic photometry were listed by Ives,[3] in his pioneering work on the photometry of lights of different colors, as follows:

"1. *The Method of Equality of Brightness.*—The two lights illuminate the two parts of a photometric field. The relative intensities of illumination from the two lights are changed until the two differently coloured fields give a sensation of equal brightness. The illuminations are then called equal.

"2. *The Method of Visual Acuity.*—Two illuminations are equal when the same fineness of detail is just distinguishable by each.

"3. *By Critical Frequency.*—Two illuminations are equal when the flicker produced by rapid alternation of one illumination with black disappears at the same speed of alternation as for the other when similarly alternated.

"4. *The Flicker Photometer.*—Two illuminations are equal when upon rapidly alternating one with the other no sensation of flicker results, the speed of alternation being such that the slightest change of either illumination produces a flicker."

Two of these methods—the method of visual acuity and the method of critical frequency—have never been used in any of the investigations where an attempt was made to obtain an average luminosity curve for a large number of observers. This is doubtless due to the lack of precision inherent in these methods.

The flicker method, on the other hand, because of its precision of setting and the reproducibility of results obtained thereby, has had extensive use as a means of determining the average luminosity curves of large numbers of observers for daylight vision. Those who have used this method for this purpose with more than 10 observers, together with the numbers of observers tested, the field sizes and retinal illuminations* used, and the dates of the measurements, are as follows:

(1) Ives,[3] 18 observers, 2° field, 150 photons,† 1912.

(2) Nutting,[4] 21 observers, 2° field, 160 photons,** 1914.

(3) Coblentz and Emerson,[12] 125 observers, 2° field, 20 photons,‡ 1917.

* The Hungarian Secretariat Committee,[57] refers to papers by König,[1] Helmholtz,[2] Ives,[3] Abney and Watson,[7] Hecht and Williams,[17] Kohlrausch,[18, 41] Rosenberg,[20] Weigel,[40] Federov and Federova,[43] Buckley,[47] Johnson,[48] Dresler,[54] Jainski,[55] and Jaggi,[56] to which may be added those by Laurens,[24] and Sloan.[29]

* The unit of retinal illumination is the "photon," representing[16] an external stimulus brightness of one candle per square meter, used with a pupillary opening of one square millimeter.

† Ives used for each observer the equivalent of an illumination of 25 metercandles (25 lux) with normal pupil. Since the flicker disk was coated with MgO, the resulting brightness may be taken as $25/\pi = 8$ candles per square meter (25 apostilbs or 2.5 millilamberts). The average pupil diameter at this brightness level is approximately 4.6 mm according to Reeves,[14] who used 6 observers, and approximately 5.6 mm according to Holladay,[28] who used 3 observers. The average for the 9 observers may, therefore, be taken as 4.9 mm, equivalent to a pupil area of 19 mm^2. The resulting retinal illumination is $8 \times 19 = 152$ photons.

** 350 metercandles, 0.57×2.57 mm ocular slit.

‡ 50 metercandles, 0.52×2.63 mm ocular slit.

(4) Reeves,[10] 13 observers, 2° field, (?) photons,* 1918.

(5) So,[15] 20 observers, 1.5° field, (?) photons,[a] 1920.

(6) Jainski,[55] 60 observers, 1.2° field, possibly 800 to 3200 photons,[b] 1938.

Ives and Coblentz and Emerson used a large surrounding field of approximately the same brightness as that of the photometric field; Nutting, Reeves, and So say nothing regarding a surrounding field. Jainski used a dark surrounding field and daylight adaptation; his results are considered further in Section IV, below. The extent of spectrum covered in the several investigations varied considerably, only Coblentz and Emerson attempting to cover the complete visible spectrum.

The equality-of-brightness method has often been used in studies of the Purkinje phenomenon or of the flicker photometer but only a few times in an attempt to obtain average data for many observers for daylight vision. Coblentz and Emerson[12] measured the relative spectral luminosity for 110 observers at a few selected wavelengths, the comparison field being illuminated by the light from a vacuum tungsten lamp. Even with this large number of observers, however, the average results were considered too uncertain to be given much weight. This uncertainty is caused, of course, by the large color difference between the spectrum light and the comparison light, and is so serious that no one has ever used the "large-step" equality-of-brightness method in an effort to establish standard average luminosity data for daylight vision.

In an effort to avoid uncertainty of this kind, recourse has sometimes been had in heterochromatic photometry to the "small-step" (step-by-step or cascade) method, wherein the large chromatic interval is divided into several smaller intervals, enabling brightnesses to be equated with considerably higher precision. The small-step equality-of-brightness method has been successfully used in two determinations of the average luminosity curve.

(1) Hyde, Forsythe, and Cady,[9] 29 observers, large field, 1918. These data were supplemented in the red and blue ends of the spectrum by data obtained by an equality-of-brightness, spectral-pyrometer method: Red end, Hyde and Forsythe,[5] 10 observers, small field, 1915; blue end, Hartman,[8] 20 observers, small field, 1918.

(2) Gibson and Tyndall,[19] 52 observers, 3° field, 1923. This investigation was undertaken at the request of the U. S. National Committee of the International Commission on Illumination and was jointly supported by the National Lamp Works of the General Electric Company and by the National Bureau of Standards.

It is not convenient in the small-step method of determining spectral luminosity to keep the brightness level constant. The retinal illuminations used by Gibson and Tyndall varied from 7 to 43 photons between 460 and 680 mμ and were less in the end regions. Those used by Hyde, Forsythe, and Cady varied from 6 photons at 500 mμ to 30 photons at 560 mμ and 18 photons at 650 mμ; in the end regions of the spectrum brightnesses were maintained at relatively high values by the pyrometer method and were considered to be such as to exclude the Purkinje phenomenon.

III. Establishment of the I.C.I. Luminosity Factors

At the conclusion of the measurements by Gibson and Tyndall, a graphical compilation was made of all existing luminosity data for daylight vision in which more than 10 observers were used, and a set of relative luminosity factors was derived and recommended for use as adequately representative of both the equality-of-brightness and the flicker methods. A total of over 200 observers (excluding duplications) was thus used in deriving these recommended average data.

The recommended spectral luminosity curve and the data on which it is based are illustrated in Fig. 1. The reasons for the choice of this particular set of recommended values are fully set forth in the original publications.[19] Fig. 1, itself, is taken from an abridged paper[20] presented at the Sixth Meeting of the International Commission on Illumination at Geneva in 1924. At that time, the Commission provisionally adopted these recommended values, as given in Table I.[21]

These values are commonly designated as the I.C.I. standard luminosity factors, and the curves

* 13 footcandles = 140 metercandles, ocular-slit dimensions not given.

[a] 50 metercandles, ocular-slit dimensions not given.

[b] See Section IV, below.

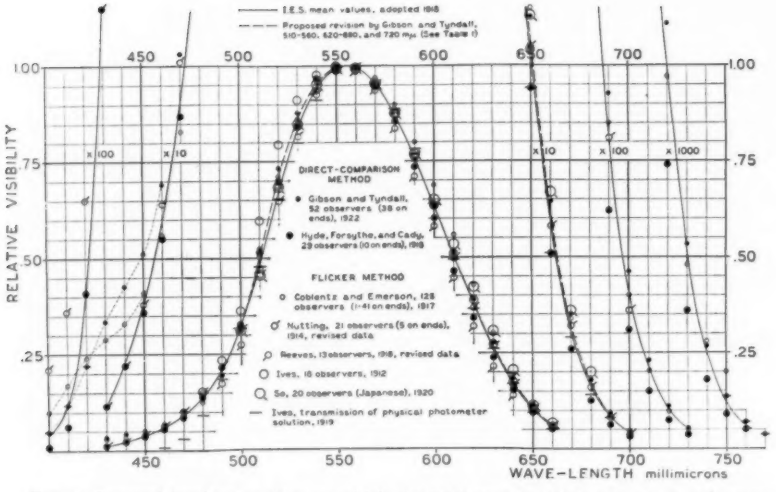

FIG. 1. Spectral luminosity data on which the 1924 standard I.C.I. luminosity factors are based. The luminosity factors adopted by the I.C.I. are those represented by the dashed line, which is identical with the continuous line over part of the spectral range. This illustration is taken from the Proceedings, 6th Session, I.C.I., 1924. (Reference 20, below.)

so defined as the I.C.I. luminosity curve. The values were accepted by the Illuminating Engineering Society and approved by the American Engineering Standards Committee as American Standard in 1925.[25] Equations approximately representative of this function were published by Tyndall and Gibson[22] in 1924 and by Walsh[26] in 1925. Interpolation of the factors to 1 mμ interval has been made by Walsh,[27] Judd,[33] and Yamauti and Okamatu,[39] by Walsh mostly to three significant figures, by the others to four.

In 1931 the I.C.I. standard luminosity factors were incorporated in the I.C.I. standard observer and coordinate system for colorimetry.[34–36] The luminosity factors serve a double purpose in this connection: (1) They define one of the three distribution functions $\bar{x}_\lambda$, $\bar{y}_\lambda$, and $\bar{z}_\lambda$, defining the chromatic properties of the I.C.I. standard observer, $\bar{y}_\lambda$ being the I.C.I. standard luminosity function. (2) Since, in the coordinate system adopted, the luminosity coefficients attached to these three distribution functions are 0, 1, and 0, respectively, the computations yielding values of the trichromatic coefficients, x, y, and z, also yield values of luminous quantities (luminous transmission, luminous apparent reflectance, etc.) directly, without additional computation. The method of choosing "primaries" such that the distribution curve for one of them coincides with the luminosity curve, was developed by Judd.[32]

In 1933 the International Committee on Weights and Measures accepted these I.C.I. luminosity factors by adoption of the following resolution, submitted by the advisory committee on electricity and photometry through its subcommittee on photometric units.

"Le Comité estime que le problème de la photométrie hétérochrome ne peut être resolu qu'en fixant *a priori* une courbe de facteurs de visibilité.

"Il suggère au Comité international des Poids et Mesures d'adopter la courbe de visibilité de Gibson et Tyndall, déjà approuvée par la Commission internationale de l'Éclairage dans son Assemblée generale de 1924.

"Cette courbe se traduit par les nombres indiqués ci-dessous.

(There is then given a table of values identical with those above.)

"Le Comité consultatif émet le voeu que le Comité international fixe les procédés pour l'utilization de cette courbe, après avis du Comité consultatif, et en accord avec les Laboratoires nationaux et la Commission internationale de l'Éclairage."

The records relating to the proposal and adoption of this resolution may be found on pp. 61–62, 65, 164, and 202–203 of the 1933 Proceedings.[37]

IV. Recent Measurements

It is apparent that the I.C.I. luminosity factors were thus in 1933 well on their way to universal adoption for practical photometry and colorimetry. In recent years, however, the validity of these values and the agreement among the small-step and large-step equality-of-brightness and the flicker methods have been questioned by various German investigators. Several papers on the subject have been published, beginning in 1935. Weigel[40] was apparently the only one of the German investigators who found the various methods to give inter-consistent results and who did not believe that the I.C.I. luminosity values should be changed. Among these investigators, emphasis was placed especially on the flicker method with 1.2° to 1.5° field ("pure cone vision") and daylight adaptation. Kohlrausch[41] disagreed with Weigel, finding results by the small-step and large-step equality-of-brightness methods to agree, but to differ notably from results obtained by the flicker method, and referred to earlier work which he had done in this connection.[18]

In 1936 Arndt[44] published results on the relative luminosity factors for the wave-lengths of the yellow and green lines of the mercury arc which disagreed notably with the values derived from the I.C.I. luminosity curve. The ratio, K_{5780}/K_{5461}, is 0.903 from the I.C.I. factors (using values interpolated either by Judd or by Yamauti and Okamatu), whereas Arndt obtained an average value of 1.035 for this ratio, using 47 normal observers, and points out that this corresponds to a shift of the wave-length of maximum luminosity from 555 mμ (I.C.I. curve) to 565 mμ. Arndt's photometric conditions were: Flicker photometry, 1.5° field, 30 lux (no artificial pupil, probably about 180 photons), 30° surrounding field. This result was confirmed by Rieck,[45] though with only 4 observers, and by Dresler[46] with 34 observers. Dresler, however, used the sodium lines instead of the mercury yellow lines. He obtained for the ratio K_{5893}/K_{5461} a value of 0.94 for a 1.2° field and 0.99 for a 7° field compared with a value of 0.777 derived from the I.C.I. factors. Dresler used a field brightness of 10 asb (probably about 45 photons),* with no surrounding field but with daylight adaptation. His value of the ratio corresponds to an increase of the wave-length of maximum luminosity by 10 to 15 mμ from the I.C.I. value.

Contrary to these results, however, are those by Preston[52] and by Forsythe, Barnes, and Shrider,[53] who have also made measurements of the ratio K_{5780}/K_{5461}. With a total of 16 observers, whose normality is indicated by their mean value of 1.003 for the Y/B ratio,† Preston obtained an average value of 0.903 for the ratio K_{5780}/K_{5460}, which is exactly the value derived from the I.C.I. values, as indicated above. He used the flicker method, 2° field, 25 to 30 lux (150 to 180 photons), with a surrounding field of similar brightness, and with the natural pupil in a dark photometer room. He found a small change in the value with

Table I. "La Commission internationale de l'Éclairage recommande, pour l'usage général, les valeurs suivantes, comme valeurs provisoires pour le facteur de visibilité.

Longueur d'onde (Mμ)	Facteur de Visibilité Relative	Longueur d'onde (Mμ)	Facteur de Visibilité Relative
400	0.0004	600	0.631
10	0012	10	503
20	0040	20	381
30	0116	30	265
40	023	40	175
450	038	650	107
60	060	60	061
70	091	70	032
80	139	80	017
90	208	90	0082
500	323	700	0041
10	503	10	0021
20	710	20	00105
30	862	30	00052
40	954	40	00025
550	995	750	00012
60	995	60	00006
70	952		
80	870		
90	757		

"Dans les cas spéciaux qui se rapportent aux régions extrêmes du spectre, ou à des conditions particulières d'étendue de champ, d'intensité, etc., ces valeurs provisoires peuvent être en défaut."

* Assuming daylight adaptation at 10 millilamberts, with pupil diameter of 4.2 mm.

† The "Y/B ratio" is the ratio of luminous transmissions of certain yellow (Y) and blue (B) solutions designed by Ives[5] to test the normality of an observer's luminosity curve for incandescent-lamp photometry. The exact compositions finally specified[13] were based on the measurements of Crittenden and Richtmyer[11] with 115 observers and give a Y/B ratio close to unity for the average normal observer with illuminant at 2080°K.

a 1° field but the difference was within experimental uncertainty. Forsythe, Barnes and Shrider, using the flicker method with 2° field, brightness above 2.5 millilamberts (above 150 photons), and with a surrounding field of comparable brightness obtained for this same ratio, K_{5780}/K_{5461}, a value 2.2 percent greater than that derived from the I.C.I. factors, or 0.923. This was the average result for 18 observers having an average Y/B ratio of 0.989.

The reasons for these discrepancies are not clear. While it is true that Preston and Forsythe, Barnes, and Shrider used the standard conditions for flicker photometry as established by Ives, which are known to give results consistent with the I.C.I. luminosity factors, and that Arndt and Dresler emphasized the importance of "pure cone vision" in their determinations, the results of both Preston and Dresler, noted above, show that differing field size will not explain the discrepancies. Furthermore, apparently, neither field brightness nor adaptive conditions will satisfactorily account for them. It is unfortunate that neither Arndt nor Dresler tested their observers by means of the Y/B ratio. Preston points out that a luminosity curve shifted as required by their data would give a Y/B ratio of 1.10.

These results are summarized in Table II, in which are included also the values obtained from Jainski's luminosity data, discussed below.

More or less consistent with the results of Arndt, Rieck, and Dresler are the determinations of the complete luminosity curve made by various observers during the past few years. These determinations have been graphically summarized by Urbanek and Ferencz in the 1939 report[57] of the Hungarian Secretariat Committee, I.C.I.* Fig. 2 is a reproduction of their Fig. 1 and shows the respective spectral luminosity data

TABLE II.

SOURCE OF DATA	NO. OF OBSERVERS	Y/B RATIO	K_{5780}/K_{5461}	K_{5893}/K_{5461}
Arndt	47	?	1.035	—
Dresler	34	?	—	0.94
Jainski	60	1.08*	1.045	0.937
Preston	16	1.003	0.903	—
Forsythe	18	0.989	0.923	—
I.C.I.	(200)	0.989*	0.903	0.777

* Computed from published spectral transmission data, reference 23, using the respective luminosity factors.

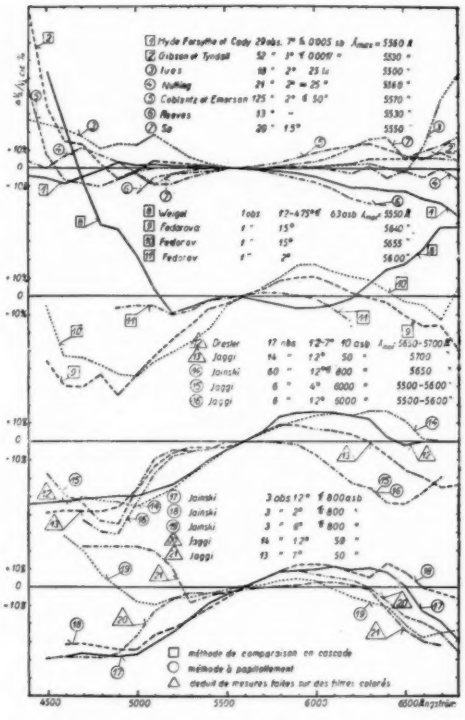

FIG. 2. Compilation of spectral luminosity data made by Urbanek and Ferencz. This figure is a reproduction of Fig. 1 of the 1939 report of the Hungarian Secretariat Committee of the I.C.I. (Reference 57, below.)

* The official summary of this report, in English, is as follows:

"SUMMARY

"Part I of the present report deals with the fundamental questions of visual photometry in general.—The fundamental problem of visual photometry could be formulated at present as follows: Examine how the adoption of the usual definition of 'visual brightness,' and the adoption of the mean visibility function as defined by the I.C.I. can ensure the required logical equivalence between the results of different visual photometric measurements, in order to establish the conditions of measurements to satisfy this equivalence in reality. That is, to decide if it would be convenient or not to change the relative visibility factors standardised at present in order to obtain a better mutual correspondence.—The problem thus formulated is treated according to the following scheme:

"*A*. Problem of the equivalence of heterochromatic brightnesses: study of the influence of the different physiological effects (Purkinje effect, etc.), and of the conditions of measurements (field brightness, etc.) on the equivalence of heterochromatic brightnesses as evaluated by the different methods. Reduction to the mean values.

"*B*. Problem of the mean visibility function: study and discussion of the results of measurements in reference to single spectral lines and the whole spectrum; and of the

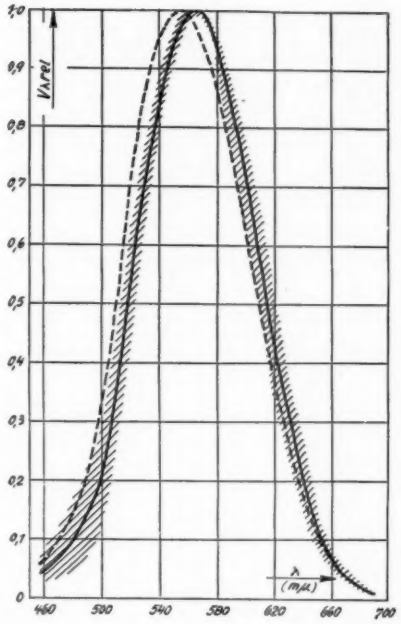

FIG. 3. Spectral luminosity data published by Jainski in 1938 (continuous line, the cross-hatching showing spread of data), compared with the standard I.C.I. (dashed) curve. This figure is a reproduction of Fig. 8 from Jainski's paper. (Reference 55, below.)

plotted in percent deviations from the standard I.C.I. luminosity factors. Curves 1 to 7 are a representation, on this basis, of the data of Fig. 1 of the present paper, the roughly symmetrical spread of the curves on either side of the zero line indicating that the I.C.I. curve is well representative of these data as a whole.

Curves 8 by Weigel,[40] and 9, 10, and 11 by Federov and Federova,[43] cannot be given much weight statistically since they apply in each case to but one observer. It is pointed out by Urbanek and Ferencz that curve 8 is almost an exact counterpart of curve 9. It may be noted that changing the field size from 1.5° to 2° brought Federov's curve into much closer agreement with the I.C.I. curve, although other observers have not found such an important effect for this difference in field size.

Of the remaining data represented in Fig. 2, it will be noted that Dresler[46] and Jaggi[56] did not make direct determinations of the spectral luminosity factors but deduced these factors from measurements made on colored filters.* Of all the recent data, therefore, only those by Jainski[55] (curve 14) represent a direct determination of the spectral luminosity function with a large number of observers. Before considering Jainski's results, however, it may be noted that (1) Dresler's data, while based on filter measurements, were also made wholly consistent with the values of K_{5893}/K_{5461} determined by him and noted above, (2) Dresler's data agree closely with those by Jainski (curve 14) in spite of the great difference in retinal illumination, (3) the data of Jaggi at moderate brightnesses with both 1.2° and 7° field size (curves 20 and 21) are intermediate (from 530 to 630 mμ) between Jainski's data (curve 14) and the I.C.I. factors, (4) the data of Jaggi (curves 15 and 16) at extremely high brightnesses, over 200 times those used for most of the data entering into the I.C.I. curve, give a narrower luminosity curve than the I.C.I. data, with again no important effect of change of field size being evident, and (5) Jainski finds no important difference in his results for field sizes of 1.2° and 2° (curves 17 and 18) but approaches the I.C.I. curve when using a 6° field (curve 19).

The data of Jainski[55] were apparently obtained with great care and with due attention to the numerous factors contributing to results of high accuracy. His spectrometer contained a second dispersing prism effectively excluding stray energy from both the photometric and radiometric measurements. The relative energy distribution at the exit slit of the monochromator was measured directly with a sensitive thermoelement. The photometric conditions of measurement were as follows: flicker photometer, field size 1.2°, field brightness 200–800 asb (retinal illumination

'Y/B ratio' data. Opinions and propositions of the different National Committees.

"*C*. Mutual correspondence of the whole of photometric results.

"Part II deals with practical questions, namely:—

"*A*. Role of visual photometry.

"*B*. Problem of the 'best method.'

"*C*. Technical questions: photometric questions, operating conditions, efficiency.

"Part III contains the resolutions proposed for discussion."

* It should be noted in this connection that Crittenden and Taylor[31] and Buckley[38] have likewise made measurements of the luminous transmissions of colored filters by the flicker method under the conditions recommended by Ives[13] and find no important differences between the results so obtained and those derived from spectrophotometric measurements and computations based on the I.C.I. luminosity factors.

possibly 800–3200 photons),† no surrounding field, daylight adaptation.

The values thus obtained for 60 observers are illustrated and compared with the I.C.I. data in Fig. 3 (Fig. 8 of Jainski's paper). Jainski comments on this comparison as follows:

(1) The maximum of the new curve is shifted from that of the I.C.I. curve by 10 mμ towards longer wave-lengths (i.e., to 565 mμ).

(2) The new average curve shows a systematic difference from the I.C.I. curve. The values fall below those of the I.C.I. curve by amounts up to 36 percent on the blue side of the maximum, and above the I.C.I. values on the red side by amounts up to 17 percent.

(3) Above 655 mμ the two curves are in agreement, while below 460 mμ the deviations are very large.

Jainski obtains from his curve a value of K_{5780}/K_{5461} equal to 1.045, in close agreement with Arndt's value of 1.035, and a value of K_{5893}/K_{5461} equal to 0.937, practically the same as Dresler's value of 0.94; see Table II. The Y/B ratio, computed by the present author from Jainski's luminosity data, is 1.085. Since the value of this ratio computed from the I.C.I. factors with the same spectral transmission data[23] is 0.989, the increase of 10 percent checks that reported by Preston (see above) to be required by the luminosity data of Arndt and Dresler.

Here, as in the case of the results of K_{5780}/K_{5461} and K_{5893}/K_{5461}, the reasons for the discrepancies are not clear. While Jainski used a smaller field and greater brightnesses than were used in obtaining the data entering into the I.C.I. curve, it should be noted that (1) he, himself, found little difference in the results for 1.2° and 2° fields (nearly all of the flicker data entering into the I.C.I. curve were obtained with a 2° field), and (2) the higher values of K_λ with high brightnesses obtained by Jainski in the red by the flicker method are contrary to the well-established inverse Purkinje effect‡ pointed out by Ives[3] and confirmed by the recent work of Jaggi[56] and Dresler.[54] Further discussion at this point, however, is unnecessary in view of the conclusions given in the following section.

V. Present Status

As a result of the above measurements, particularly those made in Germany, the German National Committee in 1937 expressed the opinion that the end to be attained should be a luminosity curve characteristic of pure cone vision and that the standard I.C.I. luminosity factors were far from satisfying that condition. It emphasized at the same time the superiority of the flicker method and proposed a redetermination of the luminosity curve and a careful study of the discrepancies existing between the recent German results and the data on which the I.C.I. curve are based.

In reply, Mr. E. C. Crittenden submitted the views of the United States National Committee in a communication* to the German National Committee, from which the following is quoted:

"Our Committee appreciates the interesting work reported by Dr. Dresler and others, but we feel that the observations reported do not logically indicate that it is necessary or desirable to adopt new luminosity factors to replace those adopted by the International Commission on Illumination and the International Committee on Weights and Measures.

"As a general principle, our Committee believes that it is desirable to evaluate lights according to their general usefulness, so far as that is practicable. It is, of course, well known that the relative values assigned to lights differing markedly in color depend upon the conditions and the method of observation. Consequently, no one method of measurement can give values which will directly indicate the usefulness of different lights for all purposes. It is believed, however, that values determined in accordance with the present standard luminosity factors are a better indication of general usefulness than those which would be obtained under the new conditions proposed.

"The argument in the communication is based upon two assumptions which do not seem justi-

† Assuming pupil diameter at 4 mm. The daylight adaptation and dark surrounding field make it uncertain what pupil diameter to assign and how much this diameter might change with change of field brightness. It appears probable that no artificial pupil was used.

‡ Jainski, in his illustration of the shift of the luminosity curve towards the longer wave-lengths with increasing field brightnesses (his Fig. 16, comparing his average data for a 2° field with the data of Ives and of Coblentz and Emerson) overlooks the fact that the intermediate curve (Coblentz and Emerson's data) was obtained with the lowest retinal illumination. See Section II, above.

* 2 and 3 (United States, 201.16.XI.38.

fied. These are (1) that the basis for evaluating lights of different colors should be *pure cone vision*, and (2) that the standard values should be those determined by observations with the *flicker photometer*.

"The condition of pure cone vision is difficult to obtain even in laboratory measurements. It is practically never approached in the use of light, especially not with artificial illumination. The luminosity factors at present accepted might be criticized because they were determined with fields of observation too *small* to represent the conditions under which light is used, but this criticism would apply still more strongly to values obtained with a field of view restricted to one degree. It is questionable also whether the general evaluation of lamps should be based upon observations made with fields of extremely high brightness.

"The flicker procedure has notable advantages as a method of comparing lights of different colors. The rapid alternation of two different lights in the field of view, however, does not correspond to conditions of practical use, and if the flicker procedure really gave values quite different from those obtained by other methods it could not reasonably be accepted as a basis for measurements to the exclusion of other methods more closely related to ordinary conditions of use.

. . . . . .

"The flicker method is a useful secondary procedure. Further studies of the effects of variation of conditions of observation with the flicker method, such as those made by Ives and recently by Dr. Jainski at the Technische Hochschule of Berlin, appear desirable. Similar investigations of the variation in results of observations by equality of brightness might also be made, particularly because previous work has indicated that the variation of the sensitivity curve with change of field brightness is not alike with the two methods of observation.

"The somewhat discordant results which have been recently obtained with the flicker photometer indicate that further study of this method of measurement is needed, rather than that values obtained by this method should be forthwith substituted for the standard luminosity factors already in use.

"In conclusion, we would emphasize the obvious fact that a change in the accepted factors would involve much confusion, because those factors have been in use for many years and are incorporated in various tables of data, particularly those used in colorimetry, as well as in photometry. Much work has also been done in the development of physical photometers based upon the present factors. We would not feel inclined to consider a change, therefore, until there is really strong evidence that the change would be a notable improvement."

The British National Committee, in view of the discrepancies among the recent results, considered that any proposal to revise the luminosity factors was premature until the causes of the discrepancies were explained, and referred particularly to the results of Preston, Winch (unpublished), and Forsythe, Barnes, and Shrider and to the average values for the Y/B ratio (1.004 to 1.017) obtained by the National Physical Laboratory during the period 1928–1938.

The Dutch National Committee saw no good reason for changing the standard I.C.I. luminosity factors; and the Swiss National Committee was definitely against any such revision, either now or in the future, particularly because of the wide usage of the standard factors in photometry and colorimetry.

In summarizing the above facts and opinions in the report of the Hungarian secretariat committee, from which the present writer has obtained much of his information, Messrs. Urbanek and Ferencz conclude: (1) There exists no average luminosity curve which is a function solely of wave-length. (2) It is impossible to eliminate completely the effect of variables other than wave-length. The use of "pure cone vision" is no remedy. (3) The average luminosity curve must be defined arbitrarily to a certain extent. (4) The definition of values should be such that it is not at variance with the ensemble of the most consistent results.

In considering whether the I.C.I. curve satisfies this last condition, Urbanek and Ferencz state that there might be, in principle, an advantage in making certain changes, not very important, in the I.C.I. values of K_λ, but that the practical value of the curve actually standardized and in current use is such that one should prefer to use, with the purpose of increasing the degree of coherence and conformity to these I.C.I. values,

corrective devices (e.g., correction filters or a method combining filters and flicker) than to change the standard curve, especially since there seem to exist no conditions more suitable as a basis for another curve.

Fortunately, the views of the German National Committee changed considerably between 1937 and 1939—in large measure due apparently to the recent conclusions of Dresler[54] that results obtained with the flicker photometer and very small field (1° 10′) do not strictly obey the additivity law. (See also Urbanek and Ferencz (reference 57, p. 15).) In a letter to the author, dated June 26, 1939, Dr. Dresler summarized the views of himself and his German colleagues on this question, and has kindly authorized their publication, as follows:

(1) No unique values of K_λ can be obtained by the flicker method for any single observer because there exists above 600 mμ a strong dependence of results on brightness, the so-called inverse Purkinje effect.

(2) Decreasing the field size below 2° is no help in this connection.

(3) Whether results by the small-step method are similarly dependent on brightness has not yet been investigated.

(4) The 1924 I.C.I. values of K_λ, derived on the basis of the American measurements are, in general, neither more incorrect nor more correct than other curves which have recently been obtained under definite modifications of the technique of measurement. Change of the present I.C.I. values is, therefore, unwarranted.

(5) Furthermore, unpublished measurements have shown that the spectral luminosity, measured between 546 and 589 mμ, is subject to time-of-year variation. In summer, the maximum luminosity is at shorter wave-lengths than in winter.

(6) Under these circumstances extremely exact control is necessary for precision measurements by subjective (visual) methods before one can be sure that a procedure yields results truly in accord with the defining equation of brightness, $B = K_m \int E_\lambda K_\lambda d\lambda \cdots$.

(7) The development of objective precision methods (for example the artificial precision eye of König[49] with the thermoelement or photo-cell) appears very urgent in this connection.

As a result, the opposition to the I.C.I. luminosity factors as expressed in the 1937 report of the German National Committee had disappeared by the time of the 1939 meetings of the International Commission on Illumination and the following recommendations were officially adopted:[59]

"Tout résultat de mesure photométrique doit être placé . . . sur la base fondamentale unique définie par la relation

$$B = K_m \int E_\lambda V_\lambda d\lambda$$

où B signifie la brillance lumineuse, K_m la valeur maximum du coefficient de visibilité, E_λ les brillances énergétiques par unité de longuer d'onde, V_λ les facteurs de visibilité relative définis par la C.I.E., λ la longuer d'onde. (Note: le symbole V_λ n'est pas adopté officiellement. . . .)

"Il n'y a pas lieu de changer les facteurs de visibilité relative actuellement en vigueur."

This decision, in agreement with the action taken by the International Committee on Weights and Measures in 1933,* officially and without qualification puts the photometry of lights of different chromaticities on the basis of the 1924 standard I.C.I. luminosity factors and clears the way for establishment of the new system of units, wherein the brightness of the blackbody at the freezing temperature of platinum is to be taken as 60 "new" candles per cm².[58]

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Supra-Threshold Visibility*

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IN various fields of visual science, certain specific systems of notation and scales of values have been devised for the appraisal of visual sensation. For example, in ophthalmology a scale of visual efficiency has been adopted by the American Medical Association[1] for expressing the visual ability of a given individual as a percentage of socalled normal vision. A similar means for expressing the attribute of "visibility" of objects of regard and of all visual tasks is likewise desirable for researches and practices which are concerned with vision and seeing. A rational scale of visibility and the procedures for expressing the visibility of any object or task upon this scale are presented herein.

Throughout this discussion, the term *visibility* is used to denote the intensity of a psychophysical stimulus which evokes visual perception and discrimination. This quantity, by definition, depends not only upon the physical characteristics of the object of regard but also upon the efficacy of the visual processes of the observer. If it be specified that all measurements of visibility are to be made by observers possessing normal vision and responses, the term visibility then denotes an attribute of the object of regard. In everyday seeing, it denotes the visual summation of various and usually complex physical characteristics of the object or task. The integration of these variables is readily accomplished within the visual centers of the brain, whereas a mathematical integration appears to be all but impossible, except in the case of the simplest of objects. However, the determination of visibility by observation with the unaided eyes is limited to threshold conditions insofar as *quantitative* values are concerned.

If an object is visible, it is necessary but not sufficient that the size of the object, its contrast with its background, the brightness of the background and the duration of the image upon the retina each exceed their individual liminal values.[2] It should be obvious that the liminal value of one factor is determined by the magnitudes of the other variables of the threshold stimulus. Each of these fundamental characteristics of the visual stimulus may be made a parameter in a relationship involving the remaining factors as constants and variables. For example, the threshold curve of Fig. 1 involves brightness as a parameter and the size and contrast of the object as variables, with the duration of the retinal image as a constant. Similar relationships involving other factors as parameters have been presented elsewhere.[2] These relationships, which we have determined with precision, provide rational quantitative bases for the development of a scale of supra-threshold visibility.

If it is assumed, for example, that the factors of brightness and contrast are constant, the visibility of a given object may be measured by the *differential* between its actual size and that size which would afford threshold visibility. This differential would be indicated on Fig. 1 by a horizontal leftward projection from any point, such as A, to the curve of threshold visibility. If this differential is small, the visibility of the object is correspondingly low. It should also be obvious that a similar evaluation of the visibility of a given object may be based upon the difference between the actual contrast and the threshold contrast of the object. This differential would be indicated on Fig. 1 by a *vertical* downward projection from any point, such as A, to the curve of threshold visibility. This concept of supra-threshold visibility is somewhat analogous to the concept of "factor of safety," as used in engineering. Actually we are dealing with objects of supra-threshold visibility. Their "elevation" above their corresponding thresholds determines visual efficacy, certainty

* Invited paper presented at the twenty-fourth annual meeting of the Optical Society of America held at the Lake Placid Club, Essex County, New York, October 13, 1939.

[1] Report of Committee on Compensation for Eye Injuries, Section on Ophthalmology, American Medical Association (1925) 368–377.

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of seeing, ease of seeing, etc., under the actual conditions of seeing.

In thus appraising visibility, it should be obvious that the "size" of the object must be expressible in a specific manner. This is practically possible only for extremely simple objects involving a single critical dimension. Hence, it follows that the development of a scale of visibility necessarily involves elementary test objects as standards. Since most objects in the world of seeing are complex in a configuration, some means must be found to compare them in threshold visibility with standard test objects of equivalent visibility. This may be accomplished with the Luckiesh-Moss visibility meter.

In developing a quantitative scale of visibility, it is recognized that the specification of a unit of visibility is an arbitrary matter. However, this is true with other units such as the meter, gram, decibel, lambert, etc. The fact that a unit of visibility may necessarily involve many arbitrary specifications does not lessen its significance as a relative measure even though the measurements themselves may be of limited significance in an absolute sense. In brief, a unit of visibility is necessarily a complicated one since it involves both biological and physical variables in a complex chain extending from the physical world to sensation and seeing. Some of the variables which have been considered in the development of our scale of visibility are presented diagrammatically in Fig. 2. In order to trace our procedures through this maze of

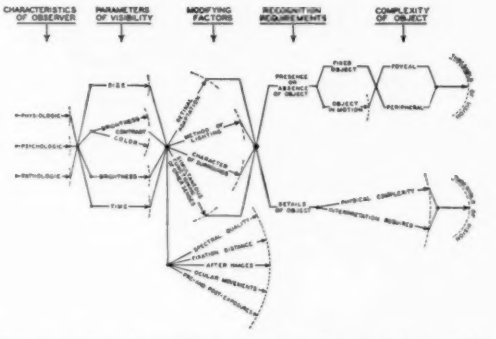

FIG. 2. Illustrating some of the major variables which influence visibility.

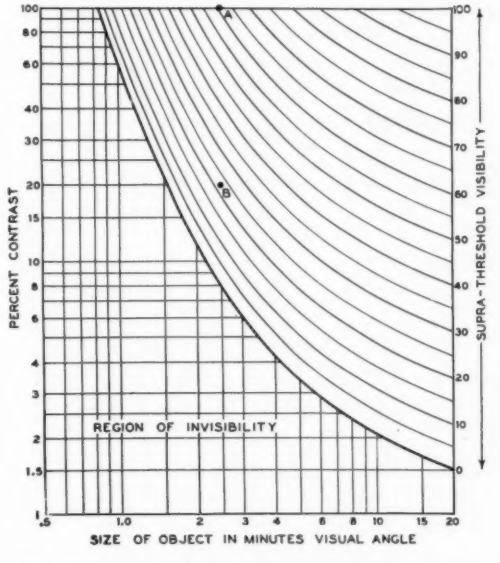

FIG. 1. The threshold relationship between size and contrast under a brightness level of 10 millilamberts and for an exposure-time equivalent to the average fixational-pause. The area below the curve of threshold visibility represents the region of invisibility. The other curves represent various degrees of supra-threshold visibility. The values at the right represent the primary scale of supra-threshold visibility.

variables, some of the basic conditions are: (1) All observations made by subjects possessing so-called normal vision and responses, (2) threshold conditions obtained chiefly by the variable of contrast, (3) all measurements made under constant and uniform brightnesses of the field of view, and (4) a criterion of threshold visibility involving foveal resolution of details of an elementary, stationary test object with a prescribed degree of certainty. The threshold relationship between size and contrast shown in Fig. 1 was determined under these specified conditions and this relationship constitutes the basis for the derivation of our scale of supra-threshold visibility.

THE LUCKIESH-MOSS SCALE OF VISIBILITY

This scale is correlated with the visibilities of a continuous series of test objects of identical configuration which vary in size from the smallest visible to the largest which is encompassable by the fovea of the human retina. The visibilities of these two objects, respectively, represent the limits of our scale of supra-threshold visibility

FIG. 3. Reproductions of the parallel-bar test objects used in calibrating the L-M visibility meter and in determining the fundamental relationship between size and contrast as variables of the threshold stimulus. If these objects are viewed from a distance of 5 feet, the numerical values below the test objects correspond to the visual angles in minutes subtended by the width of the bars or of the space separation between the bars.

when observed under prescribed conditions. It will be noted from Fig. 3 that these test objects are black squares with the middle third eliminated. They involve only one primarily critical dimension: the width of the bars or of the space between them. They are about as elementary in form as is possible with "resolution" test objects. The significance of this appraisal will be emphasized by comparing these objects with the Snellen Characters of the test chart of the ophthalmologist. Furthermore, the advantage of using the parallel-bar objects, as standards of a scale of visibility, is greatly enhanced by the fact that this form of object has been used for many years in extensive and quantitative studies of visual function.[3] No comparable data and experience are available for any other object. In a general sense, our standard parallel-bar test object may be regarded as a "common denominator" for expressing the relative visibilities of an infinite variety of objects which differ in size, configuration, contrast and brightness.

In developing our primary scale of visibility, the contrast between the standard black test object and its background was maximal, the white background was illuminated to a brightness of 10 millilamberts and all measurements of visibility were made by adult subjects possessing socalled normal vision. Since the resolving power of the human eye varies with the fixational-distance,[4] the test objects were observed from a distance of 20 feet. Under these conditions, the smallest object which was barely perceived was assigned a visibility rating of zero; and the largest encompassable by the foveal area was assigned a visibility rating of 100.

Although the cardinal point of "100" on this scale of supra-threshold visibility is an arbitrary one, it nevertheless possesses considerable rationality as a standard since it indicates an object of maximal size with respect to the foveal area and of maximal contrast with respect to its background. Its practical significance, as a cardinal point on the scale of supra-threshold visibility, is emphasized by the fact that a person who could not recognize such an object would be designated as "blind" by the ophthalmologist. In brief, the upper limit in visibility on this scale denotes approximately the highest degree of visibility concomitant with maximal size and contrast under a constant brightness of 10 millilamberts. Of course, if the brightness is increased beyond 10 millilamberts, a higher degree of visibility would be indicated as the upper limit. It will be noted from Fig. 1 that the parallel-bar test object assumed to correspond to maximal visibility is of threshold visibility when its contrast is 1.5 percent. Thus the degree of supra-threshold visibility of a black object of this size on a white background is measurable by the differential in contrast between the actual black object and one of threshold visibility. This differential in contrast (98.5 percent) is correlated with 100 units on the primary scale of visibility as is shown in Fig. 1. These units of visibility represent equal percentage differentials in contrast and are indicated by equal linear distances on the logarithmic scale of contrast. Thus the sub-division of the scale of visibility is made in conformity with the well-established fact in physiological optics that visual sensation is proportional to the logarithm of the stimulus intensity.

[3] Matthew Luckiesh and Frank K. Moss, *The Science of Seeing* (D. Van Nostrand Company, New York, 1937).

[4] M. Luckiesh and Frank K. Moss, "The Dependence of Visual Acuity Upon Stimulus-Distance," J. Opt. Soc. Am. **23**, 25 (1933).

Obviously, the scale of supra-threshold visibility (Fig. 1) could be placed on any coordinate of size which might be considered to represent optimal (as contrasted with maximal) visibility in a particular phase of seeing. For example, in the present state of typographical knowledge, it is reasonable to assume that 14-point type represents a maximal size from the viewpoint of readability. Type of this size is equivalent in visibility to a standard test object of about 7 minutes visual angle when well-printed and illuminated to 10 footcandles.[3] Hence the typographer might be inclined to place the scale of supra-threshold visibility (zero to 100) upon the size-coordinate of 7 minutes in appraising the relative visibilities of various other types.

As an example of the application of the primary scale of supra-threshold visibility, it will be noted that a standard parallel-bar test object of maximal contrast and a visual size of 2.5 minutes (represented by point *A* in Fig. 1) possesses a visibility rating of 60 units. However, if the actual contrast of this object is reduced to 20 percent (represented by point *B*) it will be noted that its visibility is 22 units on the primary scale of visibility. Thus the visibility of *any* standard parallel-bar test object, under a brightness of 10 millilamberts, may be represented by some one point on the scale of supra-threshold visibility of Fig. 1; and with the aid of the L-M visibility meter, the visibility of *any* object viewed under *any* conditions by *any* observer may be expressed upon the *same* scale by ascertaining the size of a standard parallel-bar test object of equivalent threshold visibility.

The L-M Visibility Meter

Our instruments, illustrated in Figs. 4 or 5, consist essentially of two colorless circular photographic gradient filters, precise in optical density and diffusion, which may be rotated simultaneously in front of the eyes of the subject while viewing a visual object or while performing a visual task. These gradient filters not only alter the apparent brightness of the visual field due to absorption but also alter the contrast of objects within the visual field due to the slightly diffusing character (cf. "Q" factor in photography) of the photographic filters. The construction, calibration, operation and precision of the visibility meter have been described in detail elsewhere.[3]

Fig. 4. The Luckiesh-Moss visibility meter.

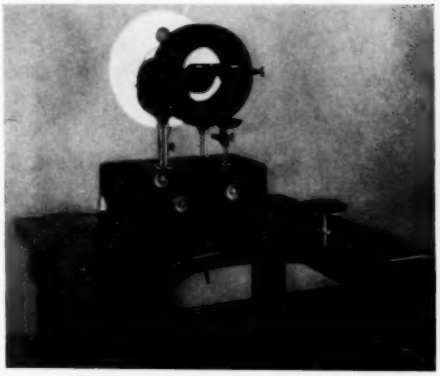

Fig. 5. The Luckiesh-Moss sensitometer (primarily intended for measuring visual efficiency) consisting of the visibility meter and certain auxiliary equipment which facilitates the operations involved in the measurement of visibility.

For the purposes of the present discussion, it is sufficient to state that the visibility meter was calibrated under the same conditions as prevailed in the determination of the size-contrast relationship of Fig. 1. In brief, subjects possessing normal vision observed (through the gradient filters of the meter) a series of parallel-bar test objects whose white backgrounds were illuminated to a constant and uniform brightness of 10 millilamberts. In each case, threshold conditions were established by rotating the gradient filters until the test object just emerged from the obscurity of the blurred visual field. Thus a value of "1," on the scale of the meter, indicates that a parallel-bar test object whose critical details subtend a visual angle of 1 minute, is of threshold visibility when viewed through that portion of the gradient filter so identified. By reference to Fig. 1 it will

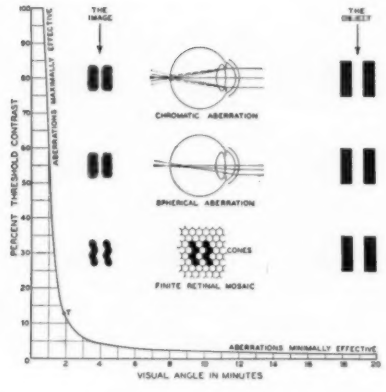

FIG. 6. The threshold relationship between size and contrast (Fig. 1) plotted on arithmetic coordinates. The diagrams illustrate the blurring of the retinal image by aberrations and the distortion of the image by the finite character of the retinal mosaic.

be noted that an object of this description is also of threshold visibility, when viewed with the naked eye, when its contrast is 60 percent. This is equivalent to a visibility rating of 12 units as will be noted from Fig. 1. In ophthalmology, the resolution of an object of this description is considered to define the limit of normal vision. However, under laboratory conditions for determining the absolute threshold, a smaller object (about 0.8 minute) may be resolved under a brightness of 10 millilamberts. Other points on the scale of the visibility meter have a similar significance in the translation of visibility meter readings to the scale of supra-threshold visibility.

Actually the lowest value on the visibility-meter scale (1.0) does not coincide with the zero value on our scale of supra-threshold visibility although the visibility meter could be calibrated for test objects of less than 1 minute in visual angle if desired. In general, such an extension of the present scale would be of little practical value except in a few isolated cases. It will also be noted that the highest value of the meter-scale (20.0) is equivalent to a visibility rating of 100 units on the primary scale of supra-threshold visibility. Obviously, these scales could be extended somewhat but such an extension would add but little to their theoretical significance and nothing to their usefulness in practice.

THE MODIFIED SCALE

In appraising the significance of a scale of visibility, defined by the smallest object visible and the largest encompassable by the fovea as limits, it is obvious that the refractive errors of the eye such as chromatic and spherical aberrations, as well as the finite characteristic of the retinal mosaic are of much greater importance in the recognition of a very small object than they are for a relatively large object. On the other hand, with large objects of low contrast with their backgrounds, the interaction of the retinal receptors is a factor of primary importance. The influence of these factors in blurring and distorting the retinal image of the object of regard is illustrated in Fig. 6. Thus it appears that for small objects of high contrast, the limit of vision is imposed by aberrations while for larger objects of low contrast, the limit of vision depends largely upon the behavior of the end-organs in the retina and upon their neural connections, as will be evident from a study of Fig. 6. Since aberrations are of minor importance for the majority of objects of regard, it appears desirable to correspondingly minimize their influence in the practical measurement of visibility.

The relationship between size and contrast shown in Fig. 1 indicates the contrast values which were used as limits for the primary scale of visibility. These values are 100 percent and 1.5 percent, respectively. However, it has been shown[5] that the range in contrast from 100 to about 13 percent represents the region in which aberrations markedly influence the threshold relationship between size and contrast. In general, a method of measuring visibility which introduces variables producing significant effects only in certain regions of the scale, will result in distorted values of visibility in other regions. Obviously, the distortion will be maximal when the zero of the scale corresponds to the visibility of the smallest object of maximal contrast which may be resolved. The distortion is minimized, on the other hand, as the object representing minimal visibility is increased in size with a

[5] M. Luckiesh and Frank K. Moss, "A View of the Cortical Integrational Process Through Liminal Visual Stimuli," J. Exp. Psychology **17**, 449 (1934).

concomitant decrease in the influence of aberrations. While the test object representing minimal visibility could be selected arbitrarily, there are certain theoretical considerations which suggest a specific size of object as a base for the modified scale of visibility.

It is evident from Fig. 6 that the influence of size decreases and that of contrast increases as the object of regard becomes larger. The point of transition from size to contrast, as the primarily important variable of the threshold stimulus, is indicated at *T* in Fig. 6. Actually, this point represents an object which requires a minimum of luminous energy for the resolution of its details.[5] It will be noted from the energy relationships of Fig. 7 that a minimum in energy is required for the threshold resolution of a parallel-bar object whose critical details subtend a visual angle of 1.9 minutes when the background brightness is 10 millilamberts. Such an object is of threshold visibility, under our standardized conditions of observation, when its contrast with its white background is 13 percent, as will be noted from Fig. 6. Thus a parallel-bar object, 1.9 minutes in size and 13 percent in contrast with its background is assumed as a minimal standard of visibility on the modified scale. This object is considered to be the smallest one which may be used as a base without the introduction of appreciable distortions due to aberrations.

The standardization of a parallel-bar test object of 1.9 minutes, as a cardinal point on the modified scale of visibility, also appears to be desirable from several empirical viewpoints. For example, the visibility of 8-point Bodoni Book type is equivalent to that of the standard basic test object when the type is illuminated to 1.5 footcandles and observed by subjects possessing normal vision.[3] Certainly this low degree of visibility represents a conservative basis for appraising the relative effectiveness of higher levels of illumination. The standard of minimal visibility is also provided by 4.5-point type when the latter is illuminated to 10 footcandles. In view of the fact that 3-point type is about the smallest that is readable under otherwise favorable conditions, and also in view of the fact that 6-point type is about the smallest actually used in practice for purposes other than brief casual reading, it appears that the base of the modified scale is a conservative one for appraising the relative effectiveness of larger types as an aid to seeing.

The modified scale of supra-threshold visibility, involving contrast limits of 13.0 percent and 1.5 percent, respectively, is divided into 100 units representing equal logarithmic intervals in contrast. Obviously, the modified scale can be represented diagrammatically in the same manner as the primary scale (Fig. 1). However, in order to obtain a higher degree of graphical precision in a chart of reasonable size, the differential between threshold visibility and maximal visibility is obtained from Fig. 8, by an arithmetical

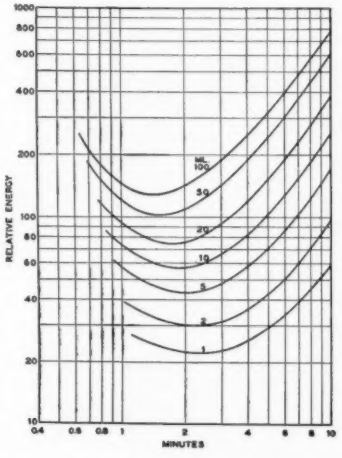

FIG. 7. The relation between relative energy and size of test object for various brightnesses of the background. All points on all curves represent conditions of threshold visibility. The left extremities of the curves represent black test objects on white backgrounds. As the object is increased in size, the contrast is decreased in order to maintain threshold conditions. The lowest points on the curves indicate the transition from size to contrast as the primarily important variable of the visual threshold.

procedure. For example, if a given object is found (with the aid of the visibility meter) to be equivalent in visibility to a standard parallel-bar test object 4.0 minutes in size, it will be noted from Fig. 8, that its supra-threshold visibility is equal to 100–51 or 49 units on the modified scale. The same object would possess a visibility-rating of about 75 units on the primary scale. It will also be noted from Fig. 1 that the modified scale includes approximately

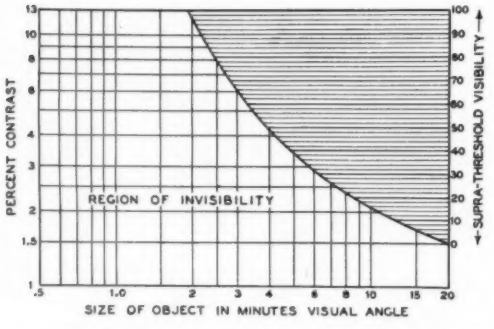

FIG. 8. The modified scale of supra-threshold visibility as applied between the size limits of 1.9 and 20.0 minutes, respectively. For graphical precision, this chart is drawn to facilitate a translation from threshold to supra-threshold visibility by arithmetic rather than by geometric means.

one-half of the primary scale of visibility. Although this relationship is an empirical one, it enhances the practical significance of the two scales.

A COMPARISON OF THE TWO SCALES OF VISIBILITY

Essentially the primary and modified scales of visibility differ in two respects: (1) the arbitrary standard of minimal visibility of the modified scale is considerably higher than the corresponding threshold limit on the primary scale; and (2) the calibration of the modified scale of visibility is not seriously influenced by the effects of aberrations. The effects of these characteristics are indicated in Table I in which the visibilities of various sizes of type are expressed upon both scales.

It will be noted from Table I that an increase in type-size from 6 to 14 points results in an increase in visibility from 63 to 86 units when measured on the primary scale of supra-threshold visibility. Thus the complete range in size of type used in practice for extensive text matter is confined to less than one-quarter of the scale from minimal to maximal visibility. However, it will be noted that the same range in type-size is indicated by a range in visibility from 26 to 72 units when expressed upon the modified scale. It is a reasonable practical assumption that the latter values indicate the relative effectiveness of type-size, as an aid to seeing, in a more appropriate manner than do the corresponding values of the primary scale. This is an important consideration from the viewpoint of extending the use of larger and more readable types through an appreciation of their effectiveness.

In some situations, such as those encountered outdoors at night, the visibilities of certain objects may be too low to be expressible upon the modified scale of supra-threshold visibility. However, comparisons of such low degrees of visibility with any reasonable standard of maximal visibility possess little, if any, practical significance. In such cases, the actual visibility-meter measurements should be adequate for designating relative visibilities.

EASE OF SEEING

Every problem of seeing eventually involves two viewpoints. Considerations of the visual object or task are primarily concerned with visibility. Of course, the human being is involved in determinations of visibility but this factor is rather definitely "attached" to the object or task. However, in the performance of seeing there is another important viewpoint—ease of seeing—which is definitely attached to the human being who is performing the task of seeing. This is particularly important in considerations of the more prolonged tasks of critical seeing. Ease of seeing is studied by measuring the psychophysiological effects of seeing. This is an important viewpoint and realm which we have invaded by many researches paralleling our work on visibility.[3] However, space permits only a glimpse from the viewpoint of ease of seeing but this is sufficient to reveal its significance and relation to visibility.

Behavioristic determinations of the relative ease with which various visual tasks are performed usually reveal that this characteristic is directly related to the visibility-rating of the task,[3] as measured by the procedures which have been described. This correlation is an extremely important one from a practical viewpoint since measurements of visibility may be made readily and with considerable precision, while appraisals of ease of seeing are not only tedious but also subject to considerable variability due to complex psychophysiological involvements. However, there are exceptions to this generalization.

For example, the visibility of a given line of print is not appreciably altered by the proximity of adjacent lines of print while "ease of reading" is markedly influenced by the space (leading) between successive lines.[6] In general, criteria of ease of seeing are often more comprehensive than those of visibility since the former involve all of the factors of visibility as well as certain psychophysiological factors whose influences may be minimized or undetected in the measurement of visibility. Usually, the visual situations in which it is advisable to apply both types of criteria will be obvious from a consideration of the characteristics and requirements of the tasks. Our researches on fatigue, tenseness, rate of blinking,[7] etc., indicate that such situations are not numerous.

TABLE I. *The basic data from which these visibility values were computed were obtained with the L-M visibility meter. All measurements were made under a diffuse illumination of 10 footcandles.*

Type-Size in Points	Equivalent Size in Minutes	Visibility	
		Primary Scale	Modified Scale
4	1.60	40	—
5	2.11	54	9
6	2.64	63	26
8	3.64	73	46
10	4.65	79	59
12	5.66	83	66
14	6.67	86	72
18	8.67	90	80
24	11.68	94	89

[6] Matthew Luckiesh and Frank K. Moss, "The Effects of Leading On Readability," J. App. Psychology **22**, 140 (1938).

[7] Matthew Luckiesh and Frank K. Moss, "Frequency of Blinking As A Criterion of Ease of Seeing," Am. J. Ophthalmology **22**, 616 (1939).

Tables of Planck's Radiation and Photon Functions*

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SEVERAL recent papers[2] on photon emission and blackbody radiation by A. G. Worthing of the University of Pittsburgh and Frank Benford of the General Electric Company have made it seem timely to compute and publish certain data on Planck's radiation function and on the photon function.

The functions under consideration are defined as follows:

$$\mathfrak{R}_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T}-1)^{-1}; \qquad \mathfrak{R}_{0-\lambda} = \int_0^\lambda \mathfrak{R}_\lambda d\lambda;$$

$$N_\lambda = 2\pi c\lambda^{-4}(e^{c_2/\lambda T}-1)^{-1}; \quad N_{0-\lambda} = \int_0^\lambda N_\lambda d\lambda,$$

where e is the base of natural logarithms.

$\mathfrak{R}_\lambda d\lambda$ is the energy emitted by a blackbody at absolute temperature T, per unit time, per unit area in a wave-length interval from λ to $\lambda+d\lambda$, throughout the solid angle 2π steradians. (The symbol $\mathfrak{R}_\lambda$ for the spectral radiancy here used supersedes the symbol J_λ frequently encountered in the literature.)

$N_\lambda d\lambda$ represents the number of photons emitted in the wave-length range from λ to $\lambda+d\lambda$, per unit time, per unit area, throughout the solid angle 2π steradians.

The integrals $\mathfrak{R}_{0-\lambda}$ and $N_{0-\lambda}$ express, respectively, the rates of emission of energy and of photons per units of area and time in a region of wave-lengths ranging from zero to λ.

The scope and arrangement of the present tables follow the recommendations of Worthing and Benford. The computations are based on the following values of the physical constants accepted after consultation with Lyman J. Briggs and H. T. Wensel[3] of the National Bureau of Standards:

$c = (2.99776 \pm 0.00020) \times 10^{10}$ cm sec.$^{-1}$
[the velocity of light],
$c_1 = (3.732 \pm 0.006) \times 10^{-5}$ erg cm^2 sec.$^{-1}$
[$= 2\pi c^2 h$, where h is Planck's constant of action],
$c_2 = (1.436 \pm 0.001)$ cm K°
[$= ch/k$, where k is Boltzmann's constant].

Table I comprises the ratios $\mathfrak{R}_{0-\lambda}/\mathfrak{R}_{0-\infty}$, $\mathfrak{R}_\lambda/\mathfrak{R}_{\lambda\,\max}$; $N_{0-\lambda}/N_{0-\infty}$, and $N_\lambda/N_{\lambda\,\max}$; ($\mathfrak{R}_{\lambda\,\max}$ and $N_{\lambda\,\max}$ are the maximum values of $\mathfrak{R}_\lambda$ and N_λ, respectively.) Since all these ratios depend on λT rather than on λ and T separately, the arguments are listed in units of λT, expressed in cm K°. They may of course be read as corresponding to $T = 1000$°K, for a suitable value of λ in microns. Thus the value of $\lambda T = 0.100$ cm K° corresponds to $\lambda = 1$ micron, $T = 1000$°K. The range of Table I is, in fact, coextensive with that of Table II, which gives $\mathfrak{R}_{0-\lambda}$, $\mathfrak{R}_\lambda$, $N_{0-\lambda}$, N_λ, for $T = 1000$°K and λ ranging between 0.50 and 20 microns.

Table III gives the values of N_λ for $T = 1000$°K, 1500°K, 2000°K, 2500°K, 3000°K, 3500°K and 6000°K, over a more limited range of λ.

CORRECTION FOR A CHANGE IN c_2 AND c_1

Let it be required to find the value of the tabulated functions based on $c_2' = rc_2$ and $c_1' = sc_1$. The corresponding corrections may be obtained in several ways. One way is to change the arguments in Table I from λT to $r\lambda T$, and the tabulated values will then be correct for the

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* Invited paper presented at the twenty-fourth annual meeting of the Optical Society of America held at the Lake Placid Club, Essex County, New York, October 13, 1939.

[1] The results reported in this paper were obtained in the course of work done by the Project for the Computation of Mathematical Tables, O.P. No. 765-97-3-10. Work Projects Administration, New York, New York, operated under the sponsorship of Dr. Lyman J. Briggs, Director of the National Bureau of Standards. The authors wish to express their appreciation to the W.P.A. and to the Sponsor of this Project for permission to publish these results.

[2] A. G. Worthing, J. Opt. Soc. Am. **28**, 176(A) (1938); Frank Benford and A. G. Worthing, Phys. Rev. **53**, 948(A) (1938); Frank Benford, "Laws and Corollaries of the Black Body," J. Opt. Soc. Am. **29**, 92 (1939); A. G. Worthing, "Radiation Laws Describing the Emission of Photons by Black Bodies," J. Opt. Soc. Am. **29**, 97 (1939); A. G. Worthing, "New λT Relation for Black Body Radiation," J. Opt. Soc. Am. **29**, 101 (1939).

[3] A discussion of the probable uncertainty in the values of the constants is given in H. T. Wensel's paper, "International Temperature Scale and Some Related Physical Constants," Nat. Bur. Stand. J. Research **22**, 375–395 (1939).

TABLE I. $\mathfrak{R}_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T}-1)^{-1}$; $\mathfrak{R}_{0-\lambda} = \int_0^\lambda \mathfrak{R}_\lambda d\lambda$; $N_\lambda = 2\pi c\lambda^{-4}(e^{c_2/\lambda T}-1)^{-1}$; $N_{0-\lambda} = \int_0^\lambda N_\lambda d\lambda$.*

	$\frac{\mathfrak{R}_{0-\lambda}}{\mathfrak{R}_{0-\infty}} = F\times10^{-p}$		$\frac{\mathfrak{R}_\lambda}{\mathfrak{R}_{\lambda\,max}} = f\times10^{-q}$				$\frac{N_{0-\lambda}}{N_{0-\infty}} = G\times10^{-p}$		$\frac{N_\lambda}{N_{\lambda\,max}} = g\times10^{-q}$			
λT IN CM K°	F	p	f	q	Δf	$\delta^2 f$	G	p	g	q	Δg	$\delta^2 g$
0.050	1.3652	9	3.1018	7	1.8320	0.6398	1.2379	10	4.7906	8	2.9818	1.0815
.051	2.2642	9	4.9339	7	2.7606	0.9286	2.0924	10	7.7724	8	4.5866	1.6048
.052	3.6788	9	7.6945	7	4.0844	1.3238	3.4638	10	1.2359	7	0.6924	0.2338
.053	5.8629	9	1.1779	6	0.5939	0.1855	5.6220	10	1.9283	7	1.0271	0.3346
.054	9.1749	9	1.7718	6	0.8497	0.2558	8.9571	10	2.9554	7	1.4983	0.4712
.055	1.4113	8	2.6216	6	1.1971	0.3474	1.4022	9	4.4537	7	2.1517	0.6534
.056	2.1358	8	3.8187	6	1.6623	0.4652	2.1590	9	6.6054	7	3.0447	0.8929
.057	3.1829	8	5.4810	6	2.2767	0.6144	3.2723	9	9.6501	7	4.2482	1.2035
.058	4.6745	8	7.7577	6	3.0781	0.8013	4.8865	9	1.3898	6	0.5849	0.1601
.059	6.7710	8	1.0836	5	0.4111	0.1033	7.1944	9	1.9747	6	0.7953	0.2104
.060	9.6798	8	1.4946	5	0.5426	0.1315	1.0451	8	2.7700	6	1.0685	0.2732
.061	1.3667	7	2.0372	5	0.7083	0.1657	1.4990	8	3.8385	6	1.4194	0.3509
.062	1.9069	7	2.7455	5	0.9150	0.2067	2.1242	8	5.2580	6	1.8654	0.4460
.063	2.6307	7	3.6606	5	1.1703	0.2553	2.9755	8	7.1234	6	2.4266	0.5612
.064	3.5907	7	4.8309	5	1.4827	0.3124	4.1225	8	9.5500	6	3.1261	0.6995
.065	4.8510	7	6.3135	5	1.8615	0.3789	5.6521	8	1.2676	5	0.3990	0.0864
.066	6.4902	7	8.1751	5	2.3172	0.4557	7.6722	8	1.6666	5	0.5048	0.1058
.067	8.6028	7	1.0492	4	0.2861	0.0544	1.0316	7	2.1714	5	0.6333	0.1285
.068	1.1302	6	1.3353	4	0.3505	0.0644	1.3744	7	2.8047	5	0.7882	0.1549
.069	1.4723	6	1.6858	4	0.4261	0.0757	1.8153	7	3.5929	5	0.9734	0.1853
.070	1.9025	6	2.1119	4	0.5145	0.0883	2.3778	7	4.5664	5	1.1935	0.2200
.071	2.4393	6	2.6264	4	0.6169	0.1024	3.0899	7	5.7599	5	1.4531	0.2596
.072	3.1045	6	3.2433	4	0.7349	0.1180	3.9847	7	7.2129	5	1.7573	0.3042
.073	3.9230	6	3.9782	4	0.8701	0.1352	5.1011	7	8.9703	5	2.1117	0.3544
.074	4.9236	6	4.8483	4	1.0240	0.1539	6.4848	7	1.1082	4	0.2522	0.0410
.075	6.1392	6	5.8723	4	1.1983	0.1743	8.1886	7	1.3604	4	0.2995	0.0472
.076	7.6070	6	7.0706	4	1.3947	0.1963	1.0274	6	1.6599	4	0.3536	0.0541
.077	9.3692	6	8.4653	4	1.6147	0.2200	1.2810	6	2.0134	4	0.4152	0.0616
.078	1.1473	5	1.0080	3	0.1860	0.0245	1.5877	6	2.4286	4	0.4850	0.0698
.079	1.3971	5	1.1940	3	0.2132	0.0272	1.9567	6	2.9136	4	0.5638	0.0788
.080	1.6923	5	1.4072	3	0.2433	0.0301	2.3981	6	3.4774	4	0.6523	0.0885
.081	2.0393	5	1.6506	3	0.2764	0.0331	2.9236	6	4.1297	4	0.7512	0.0989
.082	2.4453	5	1.9270	3	0.3127	0.0363	3.5462	6	4.8809	4	0.8613	0.1101
.083	2.9183	5	2.2397	3	0.3523	0.0396	4.2802	6	5.7421	4	0.9833	0.1220
.084	3.4668	5	2.5920	3	0.3953	0.0430	5.1418	6	6.7254	4	1.1180	0.1347
.085	4.1002	5	2.9874	3	0.4419	0.0466	6.1487	6	7.8434	4	1.2662	0.1482
.086	4.8287	5	3.4293	3	0.4922	0.0503	7.3204	6	9.1097	4	1.4287	0.1624
.087	5.6633	5	3.9215	3	0.5463	0.0541	8.6786	6	1.0538	3	0.1606	0.0177
.088	6.6159	5	4.4678	3	0.6043	0.0580	1.0247	5	1.2144	3	0.1799	0.0193
.089	7.6993	5	5.0721	3	0.6662	0.0619	1.2050	5	1.3944	3	0.2009	0.0210
.090	8.9269	5	5.7383	3	0.7322	0.0660	1.4117	5	1.5952	3	0.2235	0.0227
.091	1.0314	4	6.4705	3	0.8022	0.0700	1.6478	5	1.8188	3	0.2479	0.0244
.092	1.1874	4	7.2727	3	0.8764	0.0742	1.9165	5	2.0667	3	0.2742	0.0263
.093	1.3626	4	8.1491	3	0.9547	0.0783	2.2213	5	2.3410	3	0.3024	0.0282
.094	1.5586	4	9.1039	3	1.0372	0.0825	2.5660	5	2.6433	3	0.3325	0.0301
.095	1.7772	4	1.0141	2	0.1124	0.0087	2.9546	5	2.9758	3	0.3646	0.0321
.096	2.0204	4	1.1265	2	0.1215	0.0091	3.3914	5	3.3404	3	0.3987	0.0341
.097	2.2901	4	1.2480	2	0.1309	0.0095	3.8811	5	3.7391	3	0.4349	0.0362
.098	2.5885	4	1.3789	2	0.1408	0.0099	4.4284	5	4.1740	3	0.4733	0.0383
.099	2.9179	4	1.5197	2	0.1511	0.0103	5.0386	5	4.6473	3	0.5137	0.0404
.100	3.2804	4	1.6709	2	0.1618	0.0107	5.7171	5	5.1610	3	0.5564	0.0426

* NOTE: $\Delta f\times10^{-q}$ and $\delta^2 f\times10^{-q}$ are the first difference and the second central difference, respectively, of $f\times10^{-q}$. Similar relations hold between $\Delta g\times10^{-q}$, $\delta^2 g\times10^{-q}$ and $g\times10^{-q}$.

TABLE I—*Continued.*

λT IN CM K°	$\frac{\Re_{0-\lambda}}{\Re_{0-\infty}}=F\times10^{-p}$		$\frac{\Re_\lambda}{\Re_{\lambda\,max}}=f\times10^{-q}$				$\frac{N_{0-\lambda}}{N_{0-\infty}}=G\times10^{-}$		$\frac{N_\lambda}{N_{\lambda\,max}}=g\times10^{-q}$			
	F	p	f	q	Δf	$\delta^2 f$	G	p	g	q	Δg	$\delta^2 g$
0.100	3.2804	4	1.6709	2	0.9231	0.2664	5.7171	5	5.1610	3	3.2520	1.0668
.105	5.6770	4	2.5940	2	1.2336	0.3105	1.0346	4	8.4131	3	4.5922	1.3402
.110	9.2957	4	3.8276	2	1.5787	0.3451	1.7674	4	1.3005	2	0.6199	0.1607
.115	1.4510	3	5.4063	2	1.9464	0.3677	2.8722	4	1.9204	2	0.8049	0.1850
.120	2.1727	3	7.3527	2	2.3231	0.3767	4.4689	4	2.7254	2	1.0105	0.2056
.125	3.1370	3	9.6758	2	2.6954	0.3723	6.6929	4	3.7359	2	1.2318	0.2213
.130	4.3866	3	1.2371	1	0.3051	0.0355	9.6923	4	4.9677	2	1.4632	0.2314
.135	5.9631	3	1.5422	1	0.3378	0.0327	1.3624	3	6.4309	2	1.6990	0.2358
.140	7.9053	3	1.8800	1	0.3668	0.0290	1.8650	3	8.1299	2	1.9334	0.2344
.145	1.0248	2	2.2469	1	0.3915	0.0247	2.4934	3	1.0063	1	0.2161	0.0228
.150	1.3023	2	2.6384	1	0.4114	0.0199	3.2635	3	1.2224	1	0.2377	0.0216
.155	1.6254	2	3.0497	1	0.4262	0.0148	4.1908	3	1.4601	1	0.2577	0.0200
.160	1.9962	2	3.4759	1	0.4359	0.0098	5.2897	3	1.7179	1	0.2759	0.0181
.165	2.4161	2	3.9118	1	0.4407	0.0048	6.5733	3	1.9937	1	0.2918	0.0160
.170	2.8858	2	4.3526	1	0.4408	0.0001	8.0536	3	2.2856	1	0.3055	0.0137
.175	3.4056	2	4.7933	1	0.4365	−0.0043	9.7408	3	2.5910	1	0.3167	0.0112
.180	3.9754	2	5.2298	1	0.4282	−0.0083	1.1643	2	2.9078	1	0.3255	0.0088
.185	4.5944	2	5.6581	1	0.4165	−0.0118	1.3769	2	3.2332	1	0.3318	0.0063
.190	5.2613	2	6.0745	1	0.4017	−0.0148	1.6121	2	3.5650	1	0.3357	0.0039
.195	5.9749	2	6.4762	1	0.3843	−0.0174	1.8705	2	3.9008	1	0.3374	0.0017
.200	6.7331	2	6.8605	1	0.3648	−0.0195	2.1522	2	4.2382	1	0.3369	−0.0005
.205	7.5339	2	7.2252	1	0.3436	−0.0212	2.4573	2	4.5751	1	0.3345	−0.0025
.210	8.3750	2	7.5688	1	0.3212	−0.0224	2.7856	2	4.9096	1	0.3302	−0.0043
.215	9.2538	2	7.8900	1	0.2979	−0.0233	3.1369	2	5.2398	1	0.3243	−0.0059
.220	1.0168	1	8.1879	1	0.2740	−0.0239	3.5109	2	5.5640	1	0.3169	−0.0074
.225	1.1114	1	8.4619	1	0.2499	−0.0241	3.9071	2	5.8809	1	0.3082	−0.0087
.230	1.2091	1	8.7117	1	0.2258	−0.0241	4.3249	2	6.1891	1	0.2984	−0.0098
.235	1.3094	1	8.9375	1	0.2019	−0.0239	4.7637	2	6.4876	1	0.2877	−0.0107
.240	1.4122	1	9.1394	1	0.1785	−0.0234	5.2228	2	6.7753	1	0.2762	−0.0115
.245	1.5171	1	9.3179	1	0.1556	−0.0228	5.7015	2	7.0515	1	0.2641	−0.0121
.250	1.6239	1	9.4735	1	0.1335	−0.0221	6.1988	2	7.3156	1	0.2515	−0.0126
.255	1.7324	1	9.6071	1	0.1123	−0.0213	6.7140	2	7.5671	1	0.2385	−0.0130
.260	1.8423	1	9.7193	1	0.0919	−0.0204	7.2462	2	7.8056	1	0.2253	−0.0132
.265	1.9533	1	9.8112	1	0.0725	−0.0194	7.7944	2	8.0309	1	0.2120	−0.0134
.270	2.0653	1	9.8837	1	0.0541	−0.0184	8.3577	2	8.2429	1	0.1986	−0.0134
.275	2.1780	1	9.9377	1	0.0367	−0.0174	8.9353	2	8.4415	1	0.1852	−0.0134
.280	2.2911	1	9.9744	1	0.0204	−0.0163	9.5261	2	8.6267	1	0.1720	−0.0132
.285	2.4047	1	9.9948	1	0.0051	−0.0153	1.0129	1	8.7986	1	0.1589	−0.0131
.290	2.5183	1	9.9998	1	−0.0092	−0.0143	1.0744	1	8.9575	1	0.1461	−0.0128
.295	2.6320	1	9.9906	1	−0.0225	−0.0133	1.1369	1	9.1036	1	0.1335	−0.0126
.300	2.7454	1	9.9682	1	−0.0347	−0.0123	1.2004	1	9.2371	1	0.1212	−0.0123
.305	2.8585	1	9.9334	1	−0.0460	−0.0113	1.2648	1	9.3583	1	0.1093	−0.0119
.310	2.9712	1	9.8874	1	−0.0564	−0.0104	1.3299	1	9.4676	1	0.0978	−0.0115
.315	3.0833	1	9.8310	1	−0.0659	−0.0095	1.3958	1	9.5654	1	0.0867	−0.0111
.320	3.1947	1	9.7650	1	−0.0746	−0.0087	1.4623	1	9.6521	1	0.0759	−0.0107
.325	3.3053	1	9.6904	1	−0.0825	−0.0079	1.5294	1	9.7280	1	0.0656	−0.0103
.330	3.4150	1	9.6080	1	−0.0896	−0.0071	1.5970	1	9.7936	1	0.0557	−0.0099
.335	3.5237	1	9.5184	1	−0.0959	−0.0064	1.6650	1	9.8493	1	0.0463	−0.0095
.340	3.6314	1	9.4225	1	−0.1016	−0.0057	1.7333	1	9.8956	1	0.0372	−0.0090
.345	3.7379	1	9.3208	1	−0.1067	−0.0051	1.8020	1	9.9328	1	0.0286	−0.0086
.350	3.8432	1	9.2142	1	−0.1112	−0.0045	1.8708	1	9.9614	1	0.0204	−0.0082
.355	3.9474	1	9.1030	1	−0.1151	−0.0039	1.9399	1	9.9818	1	0.0127	−0.0078
.360	4.0502	1	8.9880	1	−0.1184	−0.0034	2.0090	1	9.9945	1	0.0053	−0.0074
.365	4.1517	1	8.8695	1	−0.1213	−0.0029	2.0782	1	9.9998	1	−0.0017	−0.0070
.370	4.2518	1	8.7482	1	−0.1238	−0.0025	2.1474	1	9.9981	1	−0.0082	−0.0066

TABLE I—*Continued.*

λT IN CM K°	$\frac{\Re_{0-\lambda}}{\Re_{0-\infty}}=F\times10^{-p}$		$\frac{\Re\lambda}{\Re\lambda\ max}=f\times10^{-q}$				$\frac{N_{0-\lambda}}{N_{0-\infty}}=G\times10^{-p}$		$\frac{N\lambda}{N\lambda\ max}=g\times10^{-q}$			
	F	p	f	q	Δf	$\delta^2 f$	G	p	g	q	Δg	$\delta^2 g$
0.375	4.3506	1	8.6244	1	−0.1258	−0.0020	2.2166	1	9.9899	1	−0.0144	−0.0062
.380	4.4479	1	8.4987	1	−0.1274	−0.0016	2.2857	1	9.9754	1	−0.0203	−0.0058
.385	4.5438	1	8.3712	1	−0.1287	−0.0013	2.3547	1	9.9551	1	−0.0257	−0.0055
.390	4.6382	1	8.2425	1	−0.1296	−0.0010	2.4236	1	9.9294	1	−0.0309	−0.0051
.395	4.7312	1	8.1129	1	−0.1303	−0.0006	2.4922	1	9.8985	1	−0.0357	−0.0048
.400	4.8227	1	7.9826	1	−0.1307	−0.0004	2.5606	1	9.8628	1	−0.0402	−0.0045
.400	4.8227	1	7.9826	1	−0.2614	−0.0015	2.5606	1	9.8628	1	−0.0845	−0.0180
.410	5.0012	1	7.7211	1	−0.2610	+0.0004	2.6966	1	9.7783	1	−0.1001	−0.0156
.420	5.1738	1	7.4601	1	−0.2590	0.0020	2.8313	1	9.6781	1	−0.1136	−0.0134
.430	5.3404	1	7.2011	1	−0.2556	0.0033	2.9645	1	9.5646	1	−0.1250	−0.0115
.440	5.5012	1	6.9455	1	−0.2512	0.0044	3.0961	1	9.4396	1	−0.1347	−0.0097
.450	5.6563	1	6.6942	1	−0.2460	0.0053	3.2258	1	9.3049	1	−0.1427	−0.0080
.460	5.8057	1	6.4483	1	−0.2401	0.0059	3.3537	1	9.1622	1	−0.1493	−0.0066
.470	5.9495	1	6.2082	1	−0.2336	0.0064	3.4795	1	9.0128	1	−0.1546	−0.0053
.480	6.0880	1	5.9746	1	−0.2268	0.0068	3.6032	1	8.8582	1	−0.1588	−0.0042
.490	6.2212	1	5.7477	1	−0.2198	0.0070	3.7248	1	8.6994	1	−0.1619	−0.0031
.500	6.3494	1	5.5279	1	−0.2126	0.0072	3.8441	1	8.5374	1	−0.1642	−0.0022
.510	6.4727	1	5.3153	1	−0.2053	0.0073	3.9611	1	8.3733	1	−0.1656	−0.0014
.520	6.5912	1	5.1100	1	−0.1980	0.0073	4.0759	1	8.2077	1	−0.1663	−0.0007
.530	6.7051	1	4.9120	1	−0.1908	0.0073	4.1884	1	8.0414	1	−0.1665	−0.0001
.540	6.8146	1	4.7212	1	−0.1836	0.0072	4.2986	1	7.8749	1	−0.1660	+0.0004
.550	6.9198	1	4.5376	1	−0.1765	0.0071	4.4065	1	7.7089	1	−0.1652	0.0009
.560	7.0209	1	4.3611	1	−0.1696	0.0069	4.5120	1	7.5437	1	−0.1639	0.0013
.570	7.1182	1	4.1915	1	−0.1629	0.0067	4.6153	1	7.3798	1	−0.1623	0.0016
.580	7.2116	1	4.0287	1	−0.1563	0.0066	4.7164	1	7.2175	1	−0.1604	0.0019
.590	7.3014	1	3.8724	1	−0.1499	0.0064	4.8152	1	7.0571	1	−0.1582	0.0022
.600	7.3877	1	3.7225	1	−0.1437	0.0062	4.9118	1	6.8989	1	−0.1558	0.0024
.600	7.3877	1	3.7225	1	−0.2815	0.0247	4.9118	1	6.8989	1	−0.3092	0.0094
.620	7.5505	1	3.4409	1	−0.2586	0.0230	5.0985	1	6.5897	1	−0.2986	0.0107
.640	7.7010	1	3.1824	1	−0.2373	0.0213	5.2768	1	6.2911	1	−0.2871	0.0115
.660	7.8402	1	2.9451	1	−0.2176	0.0196	5.4470	1	6.0040	1	−0.2752	0.0119
.680	7.9691	1	2.7275	1	−0.1996	0.0180	5.6094	1	5.7288	1	−0.2631	0.0121
.700	8.0885	1	2.5278	1	−0.1831	0.0165	5.7644	1	5.4657	1	−0.2510	0.0121
.720	8.1993	1	2.3448	1	−0.1680	0.0151	5.9122	1	5.2147	1	−0.2391	0.0119
.740	8.3020	1	2.1768	1	−0.1541	0.0138	6.0533	1	4.9756	1	−0.2274	0.0117
.760	8.3974	1	2.0226	1	−0.1415	0.0126	6.1879	1	4.7482	1	−0.2161	0.0113
.780	8.4861	1	1.8811	1	−0.1301	0.0115	6.3164	1	4.5322	1	−0.2052	0.0109
.800	8.5687	1	1.7511	1	−0.1196	0.0105	6.4390	1	4.3270	1	−0.1947	0.0105
.820	8.6455	1	1.6315	1	−0.1100	0.0096	6.5561	1	4.1323	1	−0.1846	0.0100
.840	8.7172	1	1.5215	1	−0.1013	0.0087	6.6679	1	3.9477	1	−0.1751	0.0096
.860	8.7840	1	1.4202	1	−0.0933	0.0079	6.7748	1	3.7726	1	−0.1660	0.0091
.880	8.8465	1	1.3268	1	−0.0861	0.0073	6.8769	1	3.6066	1	−0.1574	0.0086
.900	8.9048	1	1.2407	1	−0.0795	0.0066	6.9746	1	3.4492	1	−0.1491	0.0082
.920	8.9594	1	1.1613	1	−0.0734	0.0060	7.0680	1	3.3001	1	−0.1414	0.0077
.940	9.0105	1	1.0879	1	−0.0679	0.0055	7.1574	1	3.1586	1	−0.1341	0.0073
.960	9.0584	1	1.0200	1	−0.0628	0.0051	7.2430	1	3.0245	1	−0.1272	0.0069
.980	9.1033	1	9.5715	2	−0.5820	0.0463	7.3250	1	2.8974	1	−0.1206	0.0065
1.000	9.1455	1	8.9895	2	−0.5396	0.0424	7.4035	1	2.7767	1	−0.1144	0.0062
1.00	9.1455	1	8.9895	2	−1.2771	0.2660	7.4035	1	2.7767	1	−0.2754	0.0386
1.05	9.2402	1	7.7124	2	−1.0628	0.2143	7.5860	1	2.5013	1	−0.2420	0.0334
1.10	9.3217	1	6.6496	2	−0.8892	0.1736	7.7506	1	2.2594	1	−0.2132	0.0288
1.15	9.3921	1	5.7604	2	−0.7478	0.1414	7.8995	1	2.0462	1	−0.1882	0.0249
1.20	9.4532	1	5.0126	2	−0.6321	0.1157	8.0345	1	1.8580	1	−0.1666	0.0216

TABLE I—*Continued.*

λT IN CM K°	$\frac{\Re_{0-\lambda}}{\Re_{0-\infty}}=F\times10^{-p}$		$\frac{\Re_\lambda}{\Re_{\lambda\,max}}=f\times10^{-q}$				$\frac{N_{0-\lambda}}{N_{0-\infty}}=G\times10^{-p}$		$\frac{N_\lambda}{N_{\lambda\,max}}=g\times10^{-q}$			
	F	p	f	q	Δf	$\delta^2 f$	G	p	g	q	Δg	$\delta^2 g$
1.25	9.5065	1	4.3806	2	−0.5368	0.0952	8.1572	1	1.6914	1	−0.1479	0.0187
1.30	9.5531	1	3.8438	2	−0.4580	0.0788	8.2691	1	1.5435	1	−0.1316	0.0163
1.35	9.5942	1	3.3857	2	−0.3925	0.0655	8.3713	1	1.4118	1	−0.1175	0.0142
1.40	9.6304	1	2.9932	2	−0.3378	0.0547	8.4649	1	1.2944	1	−0.1051	0.0124
1.45	9.6624	1	2.6554	2	−0.2919	0.0459	8.5508	1	1.1893	1	−0.0942	0.0108
1.50	9.6909	1	2.3634	2	−0.2532	0.0387	8.6298	1	1.0951	1	−0.0847	0.0095
1.55	9.7163	1	2.1102	2	−0.2205	0.0328	8.7026	1	1.0103	1	−0.0764	0.0084
1.60	9.7390	1	1.8897	2	−0.1926	0.0278	8.7698	1	9.3394	2	−0.6899	0.0738
1.65	9.7594	1	1.6971	2	−0.1689	0.0238	8.8321	1	8.6496	2	−0.6246	0.0653
1.70	9.7777	1	1.5283	2	−0.1485	0.0204	8.8897	1	8.0249	2	−0.5667	0.0579
1.75	9.7942	1	1.3797	2	−0.1310	0.0175	8.9433	1	7.4582	2	−0.5153	0.0514
1.80	9.8091	1	1.2487	2	−0.1159	0.0151	8.9931	1	6.9429	2	−0.4695	0.0458
1.85	9.8226	1	1.1328	2	−0.1028	0.0131	9.0395	1	6.4734	2	−0.4286	0.0409
1.90	9.8349	1	1.0300	2	−0.0915	0.0114	9.0828	1	6.0448	2	−0.3920	0.0366
1.95	9.8461	1	9.3850	3	−0.8160	0.0989	9.1233	1	5.6528	2	−0.3591	0.0328
2.00	9.8563	1	8.5690	3	−0.7296	0.0864	9.1612	1	5.2937	2	−0.3297	0.0295

arguments $r\lambda T$. The interval w between successive arguments will now be changed to rw, a value less convenient for interpolation purposes than w, but still uniform.

In Table III, the values may be taken as given for $T'=rT$, and this is also valid in Table II for the functions $N_{0-\lambda}$ and N_λ. Since $\Re_\lambda$ and $\Re_{0-\lambda}$ are affected, in addition, by a change in c_1, it might be more convenient to find the corrected values $\Re_{0-\lambda}/\Re_{0-\infty}$ and $\Re_\lambda/\Re_{\lambda\,max}$ from Table I, and to multiply the results by the corrected values of $\Re_{0-\infty}$ and $\Re_{\lambda\,max}$, respectively. The latter two values may be obtained very simply for new values of the constants from the expression for these functions to be given below.

When the change in c_2 is small, it may be more convenient to use Table IV to find $d\varphi/\varphi$ and to correct as illustrated in the example given there. Here again $\Re_{0-\lambda}$ and $\Re_\lambda$ may be found by multiplying the corrected values of $\Re_{0-\lambda}/\Re_{0-\infty}$ and $\Re_\lambda/\Re_{\lambda\,max}$ by the new values of $\Re_{0-\lambda}$ and $\Re_{\lambda\,max}$, respectively.

THE FUNCTIONS $\Re_{\lambda\,max}$, $N_{\lambda\,max}$, $\Re_{0-\infty}$, $N_{0-\infty}$.

QUANTITY	VALUE	VALUE AT $T=1000$°K	λT	λ AT 1000°K
$\Re_{\lambda\,max}$	$21.20144c_1\,(T/c_2)^5$	1.2958×10^7 ergs cm^{-2} sec.$^{-1}$ μ^{-1}	$0.2014052c_2$	2.892 microns
$N_{\lambda\,max}$	$30.03263c\,(T/c_2)^4$	2.1173×10^{19} photons cm^{-2} sec.$^{-1}$ μ^{-1}	$0.2550571c_2$	3.663 "
$\Re_{0-\infty}$	$6.493939c_1\,(T/c_2)^4$	5.6994×10^7 ergs cm^{-2} sec.$^{-1}$		
$N_{0-\infty}$	$15.10611c\,(T/c_2)^3$	6.3608×10^{19} photons cm^{-2} sec.$^{-1}$		

$\Re_{\lambda\,max}$ corresponds to a value of $c_2/\lambda T=x=4.965114$, which satisfies the equation $x/1-e^{-x}=5$. Similarly $N_{\lambda\,max}$ corresponds to $x=3.920690$, satisfying the equation $x/1-e^{-x}=4$. The values $\Re_{0-\infty}$ and $N_{0-\infty}$ are obtained, respectively, from

$$\left(6\sum_{k=1}^{\infty}\frac{1}{k^4}\right)c_1(T/c_2)^4$$

and from

$$(4\pi\sum_{k=1}^{\infty}1/k^3)\cdot c(T/c_2)^3.$$

The numerical coefficients in columns 2 and 4 are universal constants, expressed to seven significant figures. In column 3, the values are given to five significant figures for purposes of computation; only three may be retained on the basis of the uncertainty in the values c_1 and c_2.

INTERPOLATION FOR λ

In Tables I and II, first differences and second central differences are given for the functions $\Re_\lambda/\Re_{\lambda\,max}$, $N_\lambda/N_{\lambda\,max}$, $\Re_\lambda$ and N_λ. Interpolation

TABLE II. $\mathfrak{R}_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T}-1)^{-1}$; $\mathfrak{R}_{0-\lambda} = \int_0^\lambda \mathfrak{R}_\lambda d\lambda$; $N_\lambda = 2\pi c\lambda^{-4}(e^{c_2/\lambda T}-1)^{-1}$; $N_{0-\lambda} = \int_0^\lambda N_\lambda d\lambda$; $T = 1000°K.$*

λ IN MICRONS	$\mathfrak{R}_{0-\lambda} = Y \times 10^p$ ERGS CM^{-2} SEC.$^{-1}$		$\mathfrak{R}_\lambda = y \times 10^q$ ERGS CM^{-2} SEC.$^{-1}$ MICRON^{-1}				$N_{0-\lambda} = Z \times 10^p$ PHOTONS CM^{-2} SEC.$^{-1}$		$N_\lambda = z \times 10^q$ PHOTONS CM^{-2} SEC.$^{-1}$ MICRON^{-1}			
	Y	p	y	q	Δy	$\delta^2 y$	Z	p	z	q	Δz	$\delta^2 z$
0.50	7.7811	−2	4.0193	0	2.3739	0.8290	1.8931	10	1.0143	12	0.6313	0.2290
0.51	1.2904	−1	6.3932	0	3.5772	1.2033	3.1999	10	1.6456	12	0.9711	0.3398
0.52	2.0967	−1	9.9705	0	5.2925	1.7153	5.2970	10	2.6167	12	1.4660	0.4949
0.53	3.3415	−1	1.5263	1	0.7696	0.2404	8.5976	10	4.0827	12	2.1745	0.7085
0.54	5.2292	−1	2.2959	1	1.1011	0.3315	1.3698	11	6.2573	12	3.1723	0.9977
0.55	8.0435	−1	3.3970	1	1.5512	0.4502	2.1444	11	9.4296	12	4.5558	1.3835
0.56	1.2173	0	4.9482	1	2.1540	0.6027	3.3017	11	1.3985	13	0.6446	0.1891
0.57	1.8140	0	7.1022	1	2.9502	0.7962	5.0043	11	2.0432	13	0.8994	0.2548
0.58	2.6642	0	1.0052	2	0.3989	0.1038	7.4727	11	2.9426	13	1.2384	0.3390
0.59	3.8591	0	1.4041	2	0.5326	0.1338	1.1002	12	4.1810	13	1.6838	0.4454
0.60	5.5169	0	1.9367	2	0.7031	0.1704	1.5983	12	5.8649	13	2.2623	0.5785
0.61	7.7893	0	2.6398	2	0.9178	0.2148	2.2924	12	8.1271	13	3.0053	0.7430
0.62	1.0868	1	3.5576	2	1.1857	0.2678	3.2485	12	1.1132	14	0.3950	0.0944
0.63	1.4994	1	4.7433	2	1.5165	0.3308	4.5504	12	1.5082	14	0.5138	0.1188
0.64	2.0465	1	6.2598	2	1.9212	0.4048	6.3044	12	2.0220	14	0.6619	0.1481
0.65	2.7648	1	8.1810	2	2.4122	0.4909	8.6436	12	2.6838	14	0.8448	0.1829
0.66	3.6990	1	1.0593	3	0.3003	0.0590	1.1733	13	3.5286	14	1.0688	0.2240
0.67	4.9031	1	1.3596	3	0.3707	0.0704	1.5775	13	4.5974	14	1.3409	0.2721
0.68	6.4417	1	1.7303	3	0.4541	0.0834	2.1018	13	5.9383	14	1.6688	0.3279
0.69	8.3915	1	2.1844	3	0.5522	0.0980	2.7761	13	7.6071	14	2.0610	0.3922
0.70	1.0843	2	2.7366	3	0.6666	0.1145	3.6363	13	9.6682	14	2.5269	0.4659
0.71	1.3903	2	3.4032	3	0.7994	0.1327	4.7253	13	1.2195	15	0.3077	0.0550
0.72	1.7694	2	4.2026	3	0.9523	0.1529	6.0937	13	1.5272	15	0.3721	0.0644
0.73	2.2359	2	5.1549	3	1.1275	0.1752	7.8011	13	1.8992	15	0.4471	0.0750
0.74	2.8062	2	6.2824	3	1.3269	0.1994	9.9171	13	2.3463	15	0.5340	0.0869
0.75	3.4990	2	7.6093	3	1.5528	0.2259	1.2523	14	2.8803	15	0.6340	0.1000
0.76	4.3356	2	9.1621	3	1.8072	0.2544	1.5711	14	3.5143	15	0.7486	0.1145
0.77	5.3399	2	1.0969	4	0.2092	0.0285	1.9589	14	4.2629	15	0.8790	0.1305
0.78	6.5389	2	1.3062	4	0.2410	0.0318	2.4280	14	5.1419	15	1.0269	0.1479
0.79	7.9628	2	1.5472	4	0.2763	0.0353	2.9923	14	6.1688	15	1.1937	0.1668
0.80	9.6451	2	1.8235	4	0.3153	0.0390	3.6674	14	7.3626	15	1.3811	0.1873
0.81	1.1623	3	2.1388	4	0.3582	0.0429	4.4710	14	8.7436	15	1.5904	0.2094
0.82	1.3937	3	2.4970	4	0.4052	0.0470	5.4231	14	1.0334	16	0.1823	0.0233
0.83	1.6633	3	2.9022	4	0.4565	0.0513	6.5456	14	1.2158	16	0.2082	0.0258
0.84	1.9759	3	3.3587	4	0.5123	0.0558	7.8632	14	1.4239	16	0.2367	0.0285
0.85	2.3369	3	3.8710	4	0.5727	0.0604	9.4030	14	1.6607	16	0.2681	0.0314
0.86	2.7521	3	4.4437	4	0.6378	0.0652	1.1195	15	1.9287	16	0.3025	0.0348
0.87	3.2278	3	5.0815	4	0.7079	0.0701	1.3272	15	2.2312	16	0.3400	0.0376
0.88	3.7707	3	5.7894	4	0.7830	0.0751	1.5670	15	2.5713	16	0.3809	0.0409
0.89	4.3881	3	6.5724	4	0.8633	0.0803	1.8428	15	2.9522	16	0.4253	0.0444
0.90	5.0879	3	7.4357	4	0.9488	0.0855	2.1589	15	3.3775	16	0.4733	0.0480
0.91	5.8781	3	8.3844	4	1.0395	0.0908	2.5199	15	3.8508	16	0.5250	0.0517
0.92	6.7678	3	9.4239	4	1.1356	0.0961	2.9308	15	4.3758	16	0.5806	0.0556
0.93	7.7661	3	1.0560	5	0.1237	0.0101	3.3969	15	4.9564	16	0.6402	0.0596
0.94	8.8831	3	1.1797	5	0.1344	0.0107	3.9241	15	5.5966	16	0.7040	0.0637
0.95	1.0129	4	1.3141	5	0.1456	0.0112	4.5184	15	6.3006	16	0.7719	0.0680
0.96	1.1515	4	1.4597	5	0.1574	0.0118	5.1864	15	7.0725	16	0.8442	0.0723
0.97	1.3052	4	1.6171	5	0.1697	0.0123	5.9353	15	7.9166	16	0.9209	0.0767
0.98	1.4753	4	1.7868	5	0.1825	0.0128	6.7723	15	8.8375	16	1.0020	0.0811
0.99	1.6630	4	1.9693	5	0.1958	0.0133	7.7055	15	9.8395	16	1.0877	0.0857
1.00	1.8696	4	2.1651	5	0.2097	0.0138	8.7431	15	1.0927	17	0.1178	0.0090

* NOTE: $10^q\Delta y$, and $10^q\delta^2 y$ are the first differences and the second central differences, respectively, of $10^q y$; similar relations hold for Δz and $\delta^2 z$ with respect to $10^q z$.

TABLE II—*Continued.*

λ IN MICRONS	$\mathfrak{R}_{0-\lambda} = Y \times 10^p$ ERGS CM^{-2} SEC.$^{-1}$		$\mathfrak{R}\lambda = y \times 10^q$ ERGS CM^{-2} SEC.$^{-1}$ MICRON^{-1}				$N_{0-\lambda} = Z \times 10^p$ PHOTONS CM^{-2} SEC.$^{-1}$		$N\lambda = z \times 10^q$ PHOTONS CM^{-2} SEC.$^{-1}$ MICRON^{-1}			
	Y	p	y	q	Δy	$\delta^2 y$	Z	p	z	q	Δz	$\delta^2 z$
1.00	1.8696	4	2.1651	5	1.1962	0.3452	8.7431	15	1.0927	17	0.6885	0.2259
1.05	3.2356	4	3.3613	5	1.5985	0.4023	1.5822	16	1.7813	17	0.9723	0.2837
1.10	5.2980	4	4.9598	5	2.0457	0.4472	2.7029	16	2.7535	17	1.3125	0.3402
1.15	8.2699	4	7.0055	5	2.5221	0.4764	4.3924	16	4.0660	17	1.7043	0.3918
1.20	1.2383	5	9.5275	5	3.0103	0.4882	6.8342	16	5.7703	17	2.1395	0.4353
1.25	1.7879	5	1.2538	6	0.3493	0.0482	1.0235	17	7.9098	17	2.6080	0.4685
1.30	2.5001	5	1.6031	6	0.3953	0.0460	1.4822	17	1.0518	18	0.3098	0.0490
1.35	3.3986	5	1.9984	6	0.4377	0.0424	2.0835	17	1.3616	18	0.3597	0.0499
1.40	4.5056	5	2.4361	6	0.4754	0.0376	2.8521	17	1.7213	18	0.4093	0.0496
1.45	5.8410	5	2.9115	6	0.5073	0.0320	3.8131	17	2.1307	18	0.4575	0.0482
1.50	7.4224	5	3.4188	6	0.5330	0.0257	4.9908	17	2.5882	18	0.5033	0.0457
1.55	9.2641	5	3.9518	6	0.5522	0.0192	6.4089	17	3.0915	18	0.5457	0.0424
1.60	1.1377	6	4.5041	6	0.5649	0.0126	8.0894	17	3.6371	18	0.5841	0.0384
1.65	1.3770	6	5.0689	6	0.5711	0.0062	1.0052	18	4.2212	18	0.6179	0.0338
1.70	1.6447	6	5.6400	6	0.5712	0.0001	1.2316	18	4.8391	18	0.6468	0.0289
1.75	1.9410	6	6.2112	6	0.5656	−0.0056	1.4896	18	5.4859	18	0.6706	0.0238
1.80	2.2658	6	6.7768	6	0.5549	−0.0107	1.7806	18	6.1565	18	0.6891	0.0186
1.85	2.6185	6	7.3317	6	0.5397	−0.0152	2.1056	18	6.8456	18	0.7025	0.0134
1.90	2.9987	6	7.8713	6	0.5205	−0.0192	2.4654	18	7.5481	18	0.7109	0.0083
1.95	3.4053	6	8.3918	6	0.4979	−0.0225	2.8605	18	8.2589	18	0.7144	0.0035
2.00	3.8375	6	8.8897	6	0.4727	−0.0253	3.2913	18	8.9733	18	0.7134	−0.0010
2.05	4.2939	6	9.3624	6	0.4452	−0.0274	3.7579	18	9.6867	18	0.7082	−0.0052
2.10	4.7733	6	9.8076	6	0.4162	−0.0291	4.2599	18	1.0395	19	0.0699	−0.0009
2.15	5.2742	6	1.0224	7	0.0386	−0.0031	4.7972	18	1.1094	19	0.0687	−0.0013
2.20	5.7951	6	1.0610	7	0.0355	−0.0031	5.3691	18	1.1780	19	0.0671	−0.0016
2.25	6.3346	6	1.0965	7	0.0324	−0.0031	5.9750	18	1.2451	19	0.0653	−0.0018
2.30	6.8911	6	1.1289	7	0.0293	−0.0031	6.6139	18	1.3104	19	0.0632	−0.0021
2.35	7.4630	6	1.1581	7	0.0262	−0.0031	7.2850	18	1.3736	19	0.0609	−0.0023
2.40	8.0487	6	1.1843	7	0.0231	−0.0030	7.9871	18	1.4345	19	0.0585	−0.0024
2.45	8.6467	6	1.2074	7	0.0202	−0.0030	8.7191	18	1.4930	19	0.0559	−0.0026
2.50	9.2556	6	1.2276	7	0.0173	−0.0029	9.4797	18	1.5489	19	0.0532	−0.0027
2.55	9.8738	6	1.2449	7	0.0145	−0.0028	1.0268	19	1.6021	19	0.0505	−0.0027
2.60	1.0500	7	1.2594	7	0.0119	−0.0026	1.1081	19	1.6526	19	0.0477	−0.0028
2.65	1.1133	7	1.2713	7	0.0094	−0.0025	1.1920	19	1.7003	19	0.0449	−0.0028
2.70	1.1771	7	1.2807	7	0.0070	−0.0024	1.2781	19	1.7452	19	0.0420	−0.0028
2.75	1.2413	7	1.2877	7	0.0048	−0.0022	1.3664	19	1.7873	19	0.0392	−0.0028
2.80	1.3058	7	1.2925	7	0.0026	−0.0021	1.4568	19	1.8265	19	0.0364	−0.0028
2.85	1.3705	7	1.2951	7	0.0007	−0.0020	1.5491	19	1.8629	19	0.0336	−0.0028
2.90	1.4353	7	1.2958	7	−0.0012	−0.0018	1.6430	19	1.8965	19	0.0309	−0.0027
2.95	1.5001	7	1.2946	7	−0.0029	−0.0017	1.7387	19	1.9275	19	0.0283	−0.0027
3.00	1.5647	7	1.2917	7	−0.0045	−0.0016	1.8357	19	1.9557	19	0.0257	−0.0026
3.05	1.6292	7	1.2872	7	−0.0060	−0.0015	1.9342	19	1.9814	19	0.0231	−0.0025
3.10	1.6934	7	1.2812	7	−0.0073	−0.0013	2.0338	19	2.0045	19	0.0207	−0.0024
3.15	1.7573	7	1.2739	7	−0.0085	−0.0012	2.1346	19	2.0252	19	0.0183	−0.0024
3.20	1.8208	7	1.2653	7	−0.0097	−0.0011	2.2363	19	2.0436	19	0.0161	−0.0023
3.25	1.8838	7	1.2557	7	−0.0107	−0.0010	2.3389	19	2.0597	19	0.0139	−0.0022
3.30	1.9463	7	1.2450	7	−0.0116	−0.0009	2.4423	19	2.0736	19	0.0118	−0.0021
3.35	2.0083	7	1.2334	7	−0.0124	−0.0008	2.5462	19	2.0853	19	0.0098	−0.0020
3.40	2.0697	7	1.2210	7	−0.0132	−0.0007	2.6508	19	2.0951	19	0.0079	−0.0019
3.45	2.1304	7	1.2078	7	−0.0138	−0.0007	2.7557	19	2.1030	19	0.0061	−0.0018
3.50	2.1904	7	1.1940	7	−0.0144	−0.0006	2.8610	19	2.1091	19	0.0043	−0.0017
3.55	2.2498	7	1.1796	7	−0.0149	−0.0005	2.9666	19	2.1134	19	0.0027	−0.0016
3.60	2.3084	7	1.1646	7	−0.0153	−0.0004	3.0723	19	2.1161	19	0.0011	−0.0016
3.65	2.3662	7	1.1493	7	−0.0157	−0.0004	3.1782	19	2.1172	19	−0.0004	−0.0015
3.70	2.4233	7	1.1336	7	−0.0160	−0.0003	3.2840	19	2.1169	19	−0.0017	−0.0014

TABLE II—*Continued.*

λ IN MICRONS	$\mathfrak{R}_{0-\lambda} = V \times 10^{p}$ ERGS CM^{-2} SEC.$^{-1}$		$\mathfrak{R}\lambda = y \times 10^{q}$ ERGS CM^{-2} SEC.$^{-1}$ MICRON^{-1}				$N_{0-\lambda} = Z \times 10^{p}$ PHOTONS CM^{-2} SEC.$^{-1}$		$N\lambda = z \times 10^{q}$ PHOTONS CM^{-2} SEC.$^{-1}$ MICRON^{-1}			
	V	p	y	q	Δy	$\delta^2 y$	Z	p	z	q	Δz	$\delta^2 z$
3.75	2.4796	7	1.1175	7	−0.0163	−0.0003	3.3898	19	2.1151	19	−0.0031	−0.0013
3.80	2.5351	7	1.1012	7	−0.0165	−0.0002	3.4955	19	2.1120	19	−0.0043	−0.0012
3.85	2.5897	7	1.0847	7	−0.0167	−0.0002	3.6010	19	2.1078	19	−0.0055	−0.0012
3.90	2.6435	7	1.0681	7	−0.0168	−0.0001	3.7063	19	2.1023	19	−0.0065	−0.0011
3.95	2.6965	7	1.0513	7	−0.0169	−0.0001	3.8112	19	2.0958	19	−0.0076	−0.0010
4.00	2.7487	7	1.0344	7	−0.0169	−0.0000	3.9158	19	2.0882	19	−0.0085	−0.0010
4.0	2.7487	7	1.0344	7	−0.0339	−0.0002	3.9158	19	2.0882	19	−0.0179	−0.0038
4.1	2.8504	7	1.0005	7	−0.0338	+0.0001	4.1238	19	2.0703	19	−0.0212	−0.0033
4.2	2.9488	7	9.6667	6	−0.3356	0.0026	4.3298	19	2.0491	19	−0.0240	−0.0028
4.3	3.0437	7	9.3311	6	−0.3313	0.0043	4.5335	19	2.0251	19	−0.0265	−0.0024
4.4	3.1354	7	8.9999	6	−0.3256	0.0057	4.7347	19	1.9986	19	−0.0285	−0.0020
4.5	3.2238	7	8.6743	6	−0.3187	0.0068	4.9332	19	1.9701	19	−0.0302	−0.0017
4.6	3.3089	7	8.3556	6	−0.3111	0.0077	5.1287	19	1.9399	19	−0.0316	−0.0014
4.7	3.3909	7	8.0445	6	−0.3027	0.0083	5.3211	19	1.9082	19	−0.0327	−0.0011
4.8	3.4698	7	7.7418	6	−0.2939	0.0088	5.5103	19	1.8755	19	−0.0336	−0.0009
4.9	3.5458	7	7.4478	6	−0.2848	0.0091	5.6962	19	1.8419	19	−0.0343	−0.0007
5.0	3.6188	7	7.1630	6	−0.2755	0.0093	5.8787	19	1.8076	19	−0.0348	−0.0005
5.1	3.6891	7	6.8875	6	−0.2660	0.0094	6.0577	19	1.7728	19	−0.0351	−0.0003
5.2	3.7566	7	6.6215	6	−0.2566	0.0095	6.2332	19	1.7378	19	−0.0352	−0.0002
5.3	3.8215	7	6.3649	6	−0.2472	0.0094	6.4052	19	1.7026	19	−0.0352	−0.0000
5.4	3.8839	7	6.1177	6	−0.2379	0.0093	6.5737	19	1.6673	19	−0.0352	+0.0001
5.5	3.9439	7	5.8798	6	−0.2287	0.0092	6.7387	19	1.6322	19	−0.0350	0.0002
5.6	4.0015	7	5.6511	6	−0.2198	0.0090	6.9002	19	1.5972	19	−0.0347	0.0003
5.7	4.0569	7	5.4313	6	−0.2110	0.0088	7.0581	19	1.5625	19	−0.0344	0.0003
5.8	4.1102	7	5.2203	6	−0.2025	0.0085	7.2127	19	1.5281	19	−0.0340	0.0004
5.9	4.1614	7	5.0178	6	−0.1943	0.0083	7.3638	19	1.4942	19	−0.0335	0.0005
6.0	4.2106	7	4.8235	6	−0.1863	0.0080	7.5115	19	1.4607	19	−0.0330	0.0005
6.0	4.2106	7	4.8235	6	−0.3648	0.0320	7.5115	19	1.4607	19	−0.0655	0.0020
6.2	4.3034	7	4.4587	6	−0.3350	0.0298	7.7971	19	1.3952	19	−0.0632	0.0023
6.4	4.3891	7	4.1237	6	−0.3075	0.0276	8.0697	19	1.3320	19	−0.0608	0.0024
6.6	4.4685	7	3.8162	6	−0.2820	0.0254	8.3300	19	1.2712	19	−0.0583	0.0025
6.8	4.5420	7	3.5342	6	−0.2587	0.0234	8.5784	19	1.2129	19	−0.0557	0.0026
7.0	4.6100	7	3.2756	6	−0.2372	0.0214	8.8154	19	1.1572	19	−0.0531	0.0026
7.2	4.6731	7	3.0383	6	−0.2176	0.0196	9.0415	19	1.1041	19	−0.0506	0.0025
7.4	4.7317	7	2.8207	6	−0.1997	0.0179	9.2572	19	1.0535	19	−0.0481	0.0025
7.6	4.7861	7	2.6209	6	−0.1834	0.0163	9.4630	19	1.0053	19	−0.0457	0.0024
7.8	4.8366	7	2.4375	6	−0.1685	0.0149	9.6595	19	9.5957	18	−0.4344	0.0231
8.0	4.8837	7	2.2690	6	−0.1549	0.0136	9.8470	19	9.1613	18	−0.4122	0.0222
8.2	4.9275	7	2.1141	6	−0.1425	0.0124	1.0026	20	8.7492	18	−0.3909	0.0212
8.4	4.9683	7	1.9715	6	−0.1313	0.0113	1.0197	20	8.3582	18	−0.3707	0.0202
8.6	5.0064	7	1.8403	6	−0.1210	0.0103	1.0361	20	7.9875	18	−0.3514	0.0193
8.8	5.0420	7	1.7193	6	−0.1116	0.0094	1.0517	20	7.6361	18	−0.3332	0.0183
9.0	5.0752	7	1.6078	6	−0.1030	0.0086	1.0666	20	7.3029	18	−0.3158	0.0173
9.2	5.1064	7	1.5048	6	−0.0951	0.0078	1.0809	20	6.9871	18	−0.2994	0.0164
9.4	5.1355	7	1.4096	6	−0.0880	0.0072	1.0946	20	6.6876	18	−0.2839	0.0155
9.6	5.1628	7	1.3217	6	−0.0814	0.0066	1.1077	20	6.4037	18	−0.2693	0.0147
9.8	5.1884	7	1.2403	6	−0.0754	0.0060	1.1202	20	6.1345	18	−0.2554	0.0138
10.0	5.2124	7	1.1649	6	−0.0699	0.0055	1.1322	20	5.8790	18	−0.2424	0.0131
10.0	5.2124	7	1.1649	6	−0.1655	0.0345	1.1322	20	5.8790	18	−0.5830	0.0818
10.5	5.2664	7	9.9936	5	−1.3772	0.2778	1.1601	20	5.2960	18	−0.5124	0.0707
11.0	5.3128	7	8.6164	5	−1.1522	0.2250	1.1853	20	4.7836	18	−0.4513	0.0611
11.5	5.3530	7	7.4643	5	−0.9690	0.1832	1.2080	20	4.3323	18	−0.3985	0.0528
12.0	5.3878	7	6.4953	5	−0.8190	0.1500	1.2287	20	3.9338	18	−0.3528	0.0457

TABLE II—*Continued.*

λ IN MICRONS	$\mathfrak{R}_{0-\lambda}=Y\times10^{p}$ ERGS CM^{-2} SEC.$^{-1}$		$\mathfrak{R}_{\lambda}=y\times10^{q}$ ERGS CM^{-2} SEC.$^{-1}$ MICRON^{-1}				$N_{0-\lambda}=Z\times10^{p}$ PHOTONS CM^{-2} SEC.$^{-1}$		$N_{\lambda}=z\times10^{q}$ PHOTONS CM^{-2} SEC.$^{-1}$ MICRON^{-1}			
	Y	p	y	q	Δy	$\delta^2 y$	Z	p	z	q	Δz	$\delta^2 z$
12.5	5.4182	7	5.6763	5	−0.6956	0.1234	1.2475	20	3.5810	18	−0.3131	0.0396
13.0	5.4448	7	4.9807	5	−0.5935	0.1021	1.2646	20	3.2679	18	−0.2787	0.0344
13.5	5.4681	7	4.3872	5	−0.5086	0.0849	1.2802	20	2.9892	18	−0.2487	0.0300
14.0	5.4888	7	3.8786	5	−0.4378	0.0709	1.2945	20	2.7405	18	−0.2225	0.0262
14.5	5.5070	7	3.4408	5	−0.3783	0.0595	1.3076	20	2.5180	18	−0.1995	0.0229
15.0	5.5233	7	3.0625	5	−0.3281	0.0501	1.3197	20	2.3185	18	−0.1794	0.0201
15.5	5.5377	7	2.7344	5	−0.2857	0.0424	1.3309	20	2.1391	18	−0.1617	0.0177
16.0	5.5507	7	2.4487	5	−0.2496	0.0361	1.3412	20	1.9774	18	−0.1461	0.0156
16.5	5.5623	7	2.1991	5	−0.2188	0.0308	1.3507	20	1.8313	18	−0.1322	0.0138
17.0	5.5727	7	1.9803	5	−0.1924	0.0264	1.3595	20	1.6991	18	−0.1200	0.0123
17.5	5.5821	7	1.7879	5	−0.1698	0.0227	1.3677	20	1.5791	18	−0.1091	0.0109
18.0	5.5906	7	1.6181	5	−0.1502	0.0196	1.3753	20	1.4700	18	−0.0994	0.0097
18.5	5.5984	7	1.4679	5	−0.1333	0.0169	1.3824	20	1.3706	18	−0.0907	0.0087
19.0	5.6054	7	1.3347	5	−0.1185	0.0147	1.3890	20	1.2798	18	−0.0830	0.0078
19.5	5.6117	7	1.2161	5	−0.1057	0.0128	1.3952	20	1.1969	18	−0.0760	0.0070
20.0	5.6175	7	1.1104	5	−0.0945	0.0112	1.4010	20	1.1208	18	−0.0698	0.0062

by the Laplace-Everett Formula will yield arithmetic accuracy considerably above what is needed on the basis of Table IV. The formula is given below:

$$\varphi(\lambda+tw)=\varphi_0+t\Delta_0+\tfrac{1}{6}(1-t)[(1-t)^2-1]\delta_0^2+\tfrac{1}{6}t(t^2-1)\delta_1^2. \quad (1)$$

Here $\varphi_0=\varphi(\lambda)$ is any of the functions whose differences are given.

Δ_0 = first difference of φ_0; namely, $\varphi(\lambda+w)-\varphi(\lambda)$; δ_0^2, δ_1^2, are the second central differences of $\varphi(\lambda)$ and $\varphi(\lambda+w)$, respectively.

For example, $\delta_0^2=\varphi(\lambda-w)-2\varphi(\lambda)+\varphi(\lambda+w)$.

Interpolation in the integrals $\mathfrak{R}_{0-\lambda}$, $N_{0-\lambda}$ in Table II and in the corresponding ratios of Table I may be made with the aid of these same differences by the following formulae:

$$F(\lambda\pm tw)=F_0\pm2.2735tw\times10^{p-q}[f_0\pm\tfrac{1}{2}t(\Delta_0-\tfrac{1}{2}\delta_0^2)+\tfrac{1}{6}t^2\delta_0^2], \quad (2)$$

$$G(\lambda\pm tw)=G_0\pm1.3845tw\times10^{p-q}[g_0\pm\tfrac{1}{2}t(\Delta_0-\tfrac{1}{2}\delta_0^2)+\tfrac{1}{6}t^2\delta_0^2], \quad (3)$$

$$Y(\lambda\pm tw)=Y_0\pm tw\times10^{q-p}[y_0\pm\tfrac{1}{2}t(\Delta_0-\tfrac{1}{2}\delta_0^2)+\tfrac{1}{6}t^2\delta_0^2], \quad (4)$$

$$Z(\lambda\pm tw)=Z_0\pm tw\times10^{q-p}[z_0\pm\tfrac{1}{2}t(\Delta_0-\tfrac{1}{2}\delta_0^2)+\tfrac{1}{6}t^2\delta_0^2]. \quad (5)$$

Here F, f, p, q, etc. have the meanings assigned to them in the table. $F_0\equiv F(\lambda)$ is the nearest tabulated value to $F(\lambda\pm tw)$ so that t is no greater than $\frac{1}{2}$, and w is the interval between two tabulated arguments. Similar definitions apply to formulae (3), (4) and (5). It may be verified that $2.2735\cdot f\cdot10^{q}$ and $1.3845\cdot g\cdot10^{q}$ are the first derivatives of $10^{p}\cdot F$ and of $10^{p}\cdot G$, respectively; the given formulae merely make use of relations between the derivatives and differences of a function. With a little practice, they are about as easy to use as ordinary difference formulae.

Formulae (4) and (5) are of course identical in form. They will give accuracy comparable with formula (1).

No differences are given in Table III, as the intervals between successive arguments are relatively large in certain parts of the table and interpolation would necessarily be cumbersome.

Example 1. Find $N_{0-\lambda}/N_{0-\infty}$ for $\lambda T=0.322$ cm K°.

Solution: In Table I the given argument lies between 0.320 cm K° and 0.325 cm K°. The corresponding entries are as follows:

T	G	p	g	q	Δg	$\delta^2 g$
0.320	1.4623	1	9.6521	1	0.0759	−0.0107
0.325	1.5294	1	9.7280	1	0.0656	−0.0103

Since 0.320 cm K° is nearest to the given argument, the differences to be used in the interpolation formula are those corresponding to

TABLE III. $N_\lambda = 2\pi c\lambda^{-4}(e^{c_2/\lambda T}-1)^{-1} = A\times 10^{p}$ *photons cm*$^{-2}$ *sec.*$^{-1}$ *micron*$^{-1}$.

λ IN MICRONS	$T=1000°K$		$T=1500°K$		$T=2000°K$		$T=2500°K$		$T=3000°K$		$T=3500°K$		$T=6000°K$	
	A	p	A	p	A	p	A	p	A	p	A	p	A	p
0.25	5.4619	0	1.1288	9	1.6229	13	5.0684	15	2.3331	17	3.5960	18	3.3543	21
0.30	3.7867	4	3.2186	11	9.3838	14	1.1251	17	2.7358	18	2.6730	19	7.9788	21
0.35	1.9064	7	1.6585	13	1.5469	16	9.3608	17	1.4428	19	1.0179	20	1.3472	22
0.40	1.8861	9	2.9691	14	1.1780	17	4.2684	18	4.6739	19	2.5830	20	1.8591	22
0.45	6.3578	10	2.6480	15	5.4040	17	1.3140	19	1.1029	20	5.0409	20	2.2618	22
0.50	1.0143	12	1.4582	16	1.7483	18	3.0898	19	2.0964	20	8.2323	20	2.5346	22
0.55	9.4296	12	5.6777	16	4.4056	18	5.9969	19	3.4192	20	1.1859	21	2.6873	22
0.60	5.8649	13	1.7099	17	9.2325	18	1.0110	20	4.9867	20	1.5599	21	2.7424	22
0.65	2.6838	14	4.2359	17	1.6829	19	1.5331	20	6.6897	20	1.9178	21	2.7246	22
0.70	9.6682	14	9.0176	17	2.7541	19	2.1429	20	8.4198	20	2.2404	21	2.6556	22
0.75	2.8803	15	1.7030	18	4.1411	19	2.8107	20	1.0086	21	2.5163	21	2.5532	22
0.80	7.3626	15	2.9212	18	5.8194	19	3.5052	20	1.1619	21	2.7409	21	2.4306	22
0.85	1.6607	16	4.6340	18	7.7425	19	4.1976	20	1.2977	21	2.9141	21	2.2976	22
0.90	3.3775	16	6.8930	18	9.8503	19	4.8638	20	1.4136	21	3.0391	21	2.1608	22
0.95	6.3006	16	9.7192	18	1.2077	20	5.4857	20	1.5089	21	3.1209	21	2.0250	22
1.00	1.0927	17	1.3103	19	1.4357	20	6.0504	20	1.5841	21	3.1649	21	1.8930	22
1.05	1.7813	17	1.7006	19	1.6632	20	6.5502	20	1.6404	21	3.1771	21	1.7668	22
1.10	2.7535	17	2.1370	19	1.8849	20	6.9815	20	1.6796	21	3.1630	21	1.6475	22
1.15	4.0660	17	2.6118	19	2.0966	20	7.3440	20	1.7034	21	3.1274	21	1.5354	22
1.20	5.7703	17	3.1167	19	2.2952	20	7.6395	20	1.7140	21	3.0749	21	1.4309	22
1.25	7.9098	17	3.6427	19	2.4782	20	7.8718	20	1.7132	21	3.0094	21	1.3337	22
1.30	1.0518	18	4.1811	19	2.6442	20	8.0456	20	1.7029	21	2.9340	21	1.2436	22
1.35	1.3616	18	4.7235	19	2.7924	20	8.1660	20	1.6846	21	2.8514	21	1.1602	22
1.40	1.7213	18	5.2624	19	2.9224	20	8.2386	20	1.6598	21	2.7640	21	1.0832	22
1.45	2.1307	18	5.7910	19	3.0345	20	8.2689	20	1.6298	21	2.6735	21	1.0121	22
1.50	2.5882	18	6.3035	19	3.1291	20	8.2621	20	1.5958	21	2.5813	21	9.4646	21
1.55	3.0915	18	6.7952	19	3.2073	20	8.2233	20	1.5586	21	2.4888	21	8.8586	21
1.60	3.6371	18	7.2621	19	3.2697	20	8.1571	20	1.5192	21	2.3968	21	8.2992	21
1.80	6.1565	18	8.8352	19	3.3857	20	7.6956	20	1.3505	21	2.0458	21	6.4550	21
2.00	8.9733	18	9.9008	19	3.3411	20	7.0609	20	1.1831	21	1.7366	21	5.0982	21
2.20	1.1780	19	1.0497	20	3.1977	20	6.3756	20	1.0297	21	1.4738	21	4.0857	21
2.40	1.4345	19	1.0713	20	3.0008	20	5.7057	20	8.9431	20	1.2543	21	3.3185	21
2.60	1.6526	19	1.0643	20	2.7804	20	5.0831	20	7.7725	20	1.0719	21	2.7286	21
2.80	1.8265	19	1.0374	20	2.5555	20	4.5204	20	6.7702	20	9.2055	20	2.2686	21
3.00	1.9557	19	9.9735	19	2.3371	20	4.0199	20	5.9154	20	7.9473	20	1.9051	21
4.00	2.0882	19	7.3946	19	1.4658	20	2.2965	20	3.1864	20	4.1125	20	8.9827	20
5.00	1.8076	19	5.2098	19	9.4064	19	1.3988	20	1.8780	20	2.3696	20	4.9089	20
6.00	1.4607	19	3.6971	19	6.2941	19	9.0566	19	1.1907	20	1.4809	20	2.9650	20
7.00	1.1572	19	2.6811	19	4.3848	19	6.1683	19	7.9934	19	9.8429	19	1.9245	20
8.00	9.1613	18	1.9915	19	3.1638	19	4.3782	19	5.6142	19	6.8629	19	1.3186	20
9.00	7.3029	18	1.5133	19	2.3520	19	3.2144	19	4.0890	19	4.9707	19	9.4237	19
10.00	5.8790	18	1.1737	19	1.7933	19	2.4271	19	3.0681	19	3.7133	19	6.9657	19

0.320 cm K°. The calculations follow:

$$w=0.005;\quad t=0.4;\quad p-q=0;$$

$$1.3845tw=0.002769;$$

$$\frac{t}{2}(\Delta_0-\tfrac{1}{2}\delta_0^2)=0.01624;\quad \tfrac{1}{6}t^2\delta_0^2=-0.00003.$$

Calculation:

g	$=9.6521$		
$\frac{1}{2}t(\Delta_0-\frac{1}{2}\delta_0^2)=0.0162$		G_0	$=1.4623$
$t^2\delta_0^2/6$	$=0$	$0.002769S=0.0268$	
$S=9.6683$		G	$=1.4891$

Hence $N_{0-\lambda}/N_{0-\infty}=10^{-p}G=0.14891$.

Example 2. Find $\mathfrak{R}_{0-\lambda}$ for $\lambda=4.17$ microns.

Solution: The relevant entries in Table II follow:

λ	Y	p	y	q	Δy	$\delta^2 y$
4.1	2.8504	7	1.0005	7	−0.0338	0.0001
4.2	2.9488	7	9.6667	6	−0.3356	0.0026

$$w=0.1;\qquad -t=-3;$$

$$q-p=-1;\quad -tw10^{q-p}=-0.003;$$

$$-\tfrac{1}{2}t(\Delta_0-\tfrac{1}{2}\delta_0^2)=-0.15(-0.3356-0.0013)$$

$$=+0.0505;$$

$$t^2\delta^2/6=0.$$

Calculation:

Y	$=9.6667$		
$-\frac{1}{2}t(\Delta_0-\frac{1}{2}\delta_0^2)$	$=0.0505$	Y_0	$=2.9488$
$t^2\delta^2/6$	$=0$	$-0.003S$	$=-0.0292$
S	$=9.7172$	Y	$=2.9196$

Hence $\mathfrak{R}_{0-\lambda}=10^{-p}\times Y=2.9196\times10^7$ ergs cm^{-2} sec.$^{-1}$.

The value S has been calculated to five significant figures for the sake of clarity in the illustration. In practice, the computation can be shortened by carrying fewer significant figures in S.

The accompanying graphs will serve to indicate the general characteristics of the tabulated functions. The authors wish to express their ap-

TABLE IV. *If ϕ is any of the functions under consideration, a change of 0.001 cm K° in c_2 will induce a corresponding change $d\phi$ in ϕ. This table lists $K=d\phi/\phi$; for a change of $\pm b\times0.001$ cm K° in c_2, the corresponding value for $d\phi/\phi$ will be approximately $\pm bK$. (The departure from this approximation will not exceed 5 units in the last tabulated place when λT is less than 0.140 cm K°, and will be less than 2 units in the last tabulated place for greater values of λT.) Example: Let it be required to find $R_{0-\lambda}/R_{0-\infty}$ for $\lambda T=0.160$ cm K°, when $c_2=1.4355$ cm K°. Solution: $b=-\frac{1}{2}$; $K=-0.00441$; $d\phi/\phi=-\frac{1}{2}bK=+0.00221$. From Table I, $\phi=0.019962$, $d\phi=(0.00221)(0.019962)=0.000044$. Hence $R_{0-\lambda}/R_{0-\infty}=0.019962+0.000044=0.02001$ to four significant figures.*

FUNCTION	$\frac{\mathfrak{R}_{0-\lambda}}{\mathfrak{R}_{0-\infty}}$	$\frac{\mathfrak{R}_\lambda}{\mathfrak{R}_{\lambda\,\max}}$	$\frac{N_{0-\lambda}}{N_{0-\infty}}$	$\frac{N_\lambda}{N_{\lambda\,\max}}$	$N_{0-\lambda}$	N_λ
λT IN CM K°	K	K	K	K	K	K
0.025						−0.040
.030						.033
.035						.029
.040						.025
.045						.022
.050	−0.0180	−0.0165	−0.0187	−0.0172	−0.0208	− .0200
.055	.0162	.0147	.0168	.0154	.0189	.0182
.060	.0147	.0132	.0153	.0139	.0174	.0167
.065	.0134	.0119	.0141	.0126	.0162	.0154
.070	.0123	.0108	.0130	.0114	.0151	.0143
.075	.0114	.00986	.0120	.0105	.0141	.0133
.080	.0105	.00902	.0112	.00971	.0133	.0125
.085	.00981	.00828	.0105	.00897	.0125	.0118
.090	.00916	.00763	.00981	.00832	.0119	.0111
.095	.00858	.00705	.00923	.00774	.0113	.0105
.100	.00806	.00652	.00870	.00721	.0108	.0100
.105	.00760	.00604	.00823	.00674	.0103	.00953
.110	.00718	.00561	.00780	.00630	.00989	.00909
.115	.00678	.00521	.00741	.00591	.00950	.00870
.120	.00643	.00485	.00706	.00555	.00914	.00834
.125	.00611	.00452	.00673	.00521	.00882	.00800
.130	.00581	.00421	.00642	.00490	.00851	.00769
.140	− .00527	− .00366	− .00588	− .00435	− .00797	− .00714
.150	.00481	.00318	.00542	.00387	.00751	.00666
.160	.00441	.00277	.00501	.00346	.00710	.00625
.170	.00406	.00240	.00465	.00309	.00674	.00588
.180	.00375	.00208	.00433	.00277	.00642	.00556
.190	.00347	.00178	.00405	.00247	.00614	.00526
.200	.00323	.00153	.00380	.00222	.00589	.00501
.250	− .00231	− .00053	− .00284	− .00122	− .00493	− .00401
.300	.00172	+ .00012	.00223	.00057	.00432	.00336
.350	.00133	.00057	.00180	.00018	.00389	.00291
.400	.00105	.00091	.00149	+ .00022	.00358	.00257
.450	.00084	.00116	.00125	.00047	.00334	.00232
.500	.00069	.00136	.00107	.00067	.00316	.00212
.550	.00057	.00152	.00093	.00083	.00302	.00196
.600	.00048	.00165	.00081	.00096	.00290	.00183
.700	− .00035	+ .00184	− .00064	+ .00115	− .00273	− .00164
.800	.00026	.00198	.00052	.00129	.00261	.00150
.900	.00020	.00209	.00043	.00140	.00252	.00139
1.000	.00016	.00217	.00036	.00148	.00245	.00131
1.500	.00006	.00240	.00018	.00171	.00227	.00108
2.000	.00003	.00251	.00011	.00182	.00220	.00097

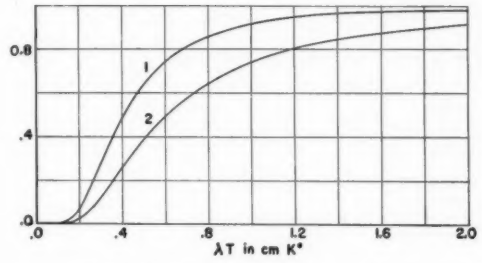

FIG. 1 A. Blackbody radiation graphs showing, as functions of λT, (1) $(1/A)\int_0^\lambda R_\lambda d\lambda$ where $A=5.6994\times10^{-5}T^4$ erg cm^{-2} sec.$^{-1}$, and (2) $(1/B)\int_0^\lambda N_\lambda d\lambda$ where $B=6.3608\times10^7T^3$ photons cm^{-2} sec.$^{-1}$.

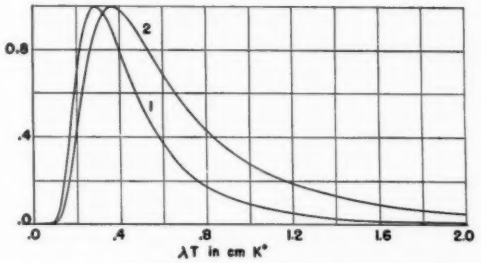

FIG. 1 B. Blackbody radiation graphs showing, as functions of λT, (1) $R_\lambda/R_{\lambda\,\max}$ and (2) $N_\lambda/N_{\lambda\,\max}$. Corresponding to $\lambda_m T=0.2892$ cm K°, $R_{\lambda\,\max}=1.2958\times10^{-5}T^5$ erg cm^{-2} sec.$^{-1}$ μ^{-1}. Corresponding to $\lambda_m T=0.3663$ cm K°, $N_{\lambda\,\max}=2.1173\times10^7T^4$ photons cm^{-2} sec.$^{-1}$ μ^{-1}.

preciation to Dr. Lyman J. Briggs, Professor A. G. Worthing, Mr. Frank Benford, and Dr. H. T. Wensel for their keen interest in the preparation of these tables, and for many valuable suggestions.

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Proceedings of the Twenty-Fourth Annual Meeting of the Optical Society of America

THE LAKE PLACID CLUB, NEW YORK, OCTOBER 12–14, 1939

THIS was the third meeting of the Society held at the Lake Placid Club. The pleasant surroundings of the Club with its opportunities for recreation and informal discussions made this meeting, like the two previous meetings held at the Club, especially enjoyable and profitable to those who attended.

The meeting was opened Thursday, October 12, at 9:30 A.M. by President Gibbs, with a session of contributed papers.

INFORMAL SOCIAL GATHERING

At an informal social gathering Thursday evening Professor Edward S. C. Smith of the Department of Geology of Union College gave an interesting illustrated lecture on "The Geological History of the Adirondack Region" explaining the many changes in the surface of the earth which had resulted in its present structure.

SYMPOSIUM OF INVITED PAPERS ON THE OPTICS OF VISION

The symposium on the Optics of Vision on Friday morning and afternoon was especially comprehensive, covering many phases of the subject, and aroused much discussion. The following papers were presented:

1. **Comparative Anatomical Studies of the Eye with Especial Reference to the Photoreceptors.**—DR. S. R. DETWILER, *College of Physicians and Surgeons, Columbia University.*
2. **The Nerve Messages in the Fibers of the Visual Pathway.**—DR. H. K. HARTLINE, *School of Medicine, University of Pennsylvania.*
3. **Chemistry of Visual Systems.**—DR. GEORGE WALD, *Biological Laboratories, Harvard University.*
4. **Recent Researches in Color Vision.**—DR. DAVID L. MACADAM, *Eastman Kodak Company, Rochester.*
5. **Aniseikonia.**—DR. GORDON H. GLIDDEN AND DR. KENNETH N. OGLE, *Dartmouth Medical School.*
6. **Luminosity Methods and Results.**—DR. KASSON S. GIBSON, *National Bureau of Standards.*
7. **The Supra-Threshold Realm of Seeing.**—DR. MATTHEW LUCKIESH AND DR. FRANK K. MOSS, *Lighting Research Laboratory, Nela Park.*

The only change from the previously announced program was the paper on color vision which was presented by Dr. MacAdam instead of Professor O'Brien. Professor O'Brien, although present, had been kept from preparing the paper he had planned by urgent personal affairs.

Dr. Detwiler reported on comparative anatomical studies of the retinas of a variety of animals, birds, and lower organisms. Particularly in the occurrence and distribution of rods and cones, there appears to be a close and natural relation between the visual capacity of animals as indicated by the retinal structure and their diurnal and nocturnal habits. For example, bats with characteristically nocturnal habits have retinas which contain only rod elements, whereas turtles have only cone elements and are not able to see at night. Most night animals, moreover, have relatively very large lenses so that they have good light-gathering power. The visual acuity of birds such as hawks seems to be accentuated by structural differences in the fovea and also by the existence of extremely fine cones. Most animals have some cones such as are associated in humans with color vision. Attention was directed to the oil drop which exists in some cones and was referred to subsequently by Dr. Wald as possibly constituting the physical basis for trichromatic vision.

Part of the mechanism of seeing is the nerve impulse which proceeds from the retina to the cortex of the brain, and the character, or mechanism of this message was reported by Dr. Hartline. The impulse has been studied by means of oscillograms of the amplified action potentials in single optic nerve fibers. It has been shown that the nerve impulse is of a single kind, that is, a given nerve fiber cannot transmit different

kinds of messages. The impulse is a rhythmic one of constant magnitude, but of increased frequency as the intensity of the stimulus is increased. The relative effectiveness of different wave-lengths of light depends upon the absorption spectrum of the photosensitive substance of the receptor cell. Responses to different wavelengths may be matched merely by adjusting the intensities of the stimuli. Usually there is a period of latency after the stimulus is applied before the response begins. Prolonged exposure results in a response whose frequency is initially high, but which soon falls to a low level, possibly because of the depletion of the photosensitive material under the action of light. The regeneration process may be studied by recording responses to a constant flash at intervals following exposure to a bright light. An increase in sensitivity of the receptor cell is thus shown to take place during dark adaptation. The path from the retina to the cortex is a complicated one, and while the nerve fibers associated directly with the retina can be studied, the next link in the chain is not generally accessible, but the third link is. The activity in neurones central to the receptor cells may be studied in the optic nerve fibers of cold-blooded vertebrates. Three types of response are established: (1) a steady discharge maintained during steady illumination; (2) bursts of impulses at the onset of illumination and at cessation; (3) a discharge only in response to cessation. These last two types of response are very sensitive to the movement of the stimulus spot on the retina. Responses in a single fiber can be elicited from a region of the retina of small but appreciable extent, so that responses are determined by numbers of receptors illuminated as well as by the intensity of illumination.

Dr. George Wald spoke on the photochemistry of the retina and showed that the parallelism of the absorption curve of rhodopsin and the so-called scotopic visibility curve is very exact if adequate corrections are made for the photochemical changes of rhodopsin on the one hand, and for the absorption of light within the eye on the other hand. This accounts for the photochemistry of the rod-like elements. It was believed that an analogous compound must be responsible for the photochemistry of the cone elements, with a similar relation to photopic vision. Such a material Dr. Wald designated as iodopsin. Skillful work involving the partial bleaching of extracts from the retina and the computing of differences in absorption has given the general character of the absorption curve of iodopsin. The cones of the retina of chickens are equipped with three groups of globules, red, golden, and greenish yellow in color, through which light must filter, and it is suggested that these may form the basis of trichromatic vision in chickens. The absorption curves of these "color filters" have been obtained and the pigments have been isolated. The rest of Dr. Wald's paper was concerned with the phenomenon of night-blindness, that is, the threshold of rod vision, which appears to be a simple inverse function of rhodopsin concentration. One of the paths by which the photodecomposition products of rhodopsin regenerate fresh rhodopsin in the retina involves vitamin A. Deficiency of vitamin A in the retina would therefore delay the formation of rhodopsin, with concurrent symptoms of night-blindness. It was shown experimentally that night-blindness would result from a vitamin A deficient diet over a period of weeks, but the cure was complete and remarkably rapid when vitamin A was administered.

Dr. MacAdam's paper on "Recent Research in Color Vision" was a discussion of coordinate systems which would adequately represent the visual appreciation of color differences, starting with the equal-chromaticity-scale triangle proposed by Judd. Using the methods of projective geometry, the restricted existing data relating to the sensibility of human vision to color differences were compared with the predictions made on the basis of transformations from the I.C.I. color diagram. Since the color diagram itself is based entirely on statements of the equivalence of stimuli and does not involve any consideration of the magnitude of color differences, the establishment of a transformation would constitute a discovery of profound importance. It is concluded, however, that the forms which have been given to the transformations so far are not sufficiently accurate and, in fact, that no linear transformation from the I.C.I. coordinate system will satisfy the requirements; i.e., the general trends indicated by transformations do not qualitatively fit the experimental data. The possibil-

ity of such a coordinate system cannot be either accepted or rejected on the basis of existing experimental data, but the discussion of such a colorimetric system is of great value in stimulating and correlating research on color differences. Commenting on this paper, Dr. O'Brien called attention to the great gap which exists between the data on color vision presented by Dr. MacAdam and the anatomical and physiological studies reported in the previous papers. The complexity of the processes involved in the simple act of seeing is so great as to have defied exact analysis in spite of the brilliant progress which has been made in related fields of study.

The paper on aniseikonia, the visual defect which consists of differences of size or shape of the ocular images of the same object seen in the two eyes in binocular vision, was presented by Drs. Glidden and Ogle, of the Dartmouth Medical School. This defect in its more acute forms results in defective vision, headaches, and even psychopathological conditions. The effect was vividly illustrated by demonstrations. The spectators were furnished with lenses which would simulate the conditions of aniseikonia and numerous objects were provided at which they could look. These included a badly skewed model of a room. With the glasses the skewed model room, in which no angle was square, looked square and square objects looked skewed. Estimates of distance were very misleading. A man standing with arms outstretched side-wise appeared to have one arm longer than the other, or if the arms were held horizontally and toward the observer, one hand appeared much larger and nearer than the other. Level surfaces seemed tipped etc. etc. It appeared abundantly apparent that a sufferer from such a visual defect would have difficulty in adjusting his visual impressions to the external world. The corrective measure is to place a distorting lens over one eye to bring the images to the same size and shape, and in many cases this has produced highly gratifying results. In special cases the lens may magnify only the horizontal or the vertical meridian. Extensive studies are being made at Dartmouth on the correlation of aniseikonia with speed of reading.

Dr. Gibson discussed data bearing on the precise form of the luminosity or "visibility" curve. As a result of extensive measurements by the flicker and equality-of-brightness methods, mostly in America, a certain set of average luminosity data was adopted by the International Commission on Illumination in 1924. These I.C.I. luminosity factors were incorporated in the 1931 I.C.I. standard observer and coordinate system for colorimetry and were accepted by the International Committee on Weights and Measures in 1933. In the period from 1935 to 1938, however, the validity of these factors was challenged by several German investigators, who believed as a result of their measurements that a new set of standard values should be adopted, based on results by the flicker photometer with pure cone vision (1.2° field and daylight adaptation). The German measurements, however, were not wholly consistent and were in conflict with similar recent measurements in England and America. At the 1939 meetings of the I.C.I., the Germans withdrew their opposition to the 1924 I.C.I. luminosity factors and the Commission formally reaffirmed their validity in the photometry of lights of different colors.

The final paper in the symposium, "The Supra-Threshold Realm of Seeing," was presented by Dr. Frank K. Moss for the joint authors and was an account of their method of establishing a scale for measuring or recording supra-threshold visibility. The term *visibility* is defined to denote the intensity of a psychophysical stimulus which evokes visual perception. The result is expressed in terms of the departure from threshold conditions. Thus dissimilar objects are assumed to be equal in visibility when the same diminution in some physical characteristic results in the threshold perception of both. The extent of the reduction required to produce threshold visibility is taken as a measure of the supra-threshold visibility of the object of regard. The Luckiesh-Moss scale of supra-threshold visibility is divided into 100 units between zero and "maximal" visibility with each unit representing an equal logarithmic increment in contrast between the limits of threshold contrast and maximal contrast, respectively, as determined for a standard parallel-bar test-object whose critical details subtend a visual angle of 20 minutes. Thus the subdivision of the scale of supra-threshold visi-

bility is made in conformity with the established principle that visual sensation (visibility) is proportional to the logarithm of the stimulus intensity. The high degree of visibility afforded by the standard test-object is emphasized by the fact that a person who could not recognize such an object would be considered as blind by the ophthalmologist. The visibility of any object may be measured by comparing it, under threshold conditions, with a continuous series of standard test-objects of known degrees of supra-threshold visibility. Obviously, the given object and some one standard test-object will be found to be equivalent in visibility. In practice, threshold conditions are readily obtained with the Luckiesh-Moss visibility meter. In addition, this instrument is so calibrated that the contrast characteristic (and hence the degree of supra-threshold visibility) of the equivalent standard test-object may be determined without actually observing the series of standard test-objects in each case. When visual tasks are performed under controllable conditions, the visibility will rarely be less than the equivalent of a 2-minute standard test-object. Hence the precision of expressing the results can be expressed by using a scale limited to the range between 2 and 20 minutes. This scale is referred to as the "modified" scale of supra-threshold visibility. Several examples of the use of both scales were presented and discussed.

Annual Dinner and Award of the Frederic Ives Medal

The feature of the Annual Dinner Friday evening was the presentation of the Frederic Ives Medal to Professor A. H. Pfund of Johns Hopkins University.

At the close of the dinner President Gibbs introduced the newly elected officers of the Society.

Talk by Professor A. H. Pfund

Following the dinner Professor Pfund gave an illustrated talk on experimental investigations of the "Optical Properties of Small Particles" to which he has made important contributions. He considered the particles "small" when their linear dimensions are small in comparison with the wave-length of light. Under these conditions Rayleigh's law of scattering may be expected to apply. Instead of studying the scattered radiations it was found more desirable to investigate transmitted light. By carrying the work into the near infra-red it was possible to show that Rayleigh's law is closely obeyed when the (pigment) particles are both small and uniform in size. From the study of such transmission curves it is now possible to evaluate particle size as well as particle-size distribution. It is even possible from such studies to gain information as to the degree of dispersion or aggregation of the pigment in a finished paint film. When the pigment particles are allowed to form small islands separated by clear spaces (mosaic films) the "colors of mixed plates" are revealed. When the technique of forming such films shall have been perfected it will be possible to determine the refractive index, particle-size and distribution from the transmission curves of such mosaic films.

A layer of fumed MgO on a plate of rocksalt may be quite opaque to visible light, yet be transparent in the infra-red. Films of this type are now known as powder filters. Metals when evaporated in a partial vacuum are deposited as intensely black films. Powder films of quartz, calcite, etc. when heated emit intense radiations of the same wave-lengths as those reflected metallically from massive crystals, i.e., in these regions the crystal reflects metallically while the corresponding powder is "black."

When the transmission of a thin layer of rather coarse particles of quartz or calcite is studied, it is found that a strong, narrow band exists on the short wave-length side of the region of metallic reflection. Such bands are as characteristic a property of crystals in the infra-red as are residual rays.

Business Session

A brief business session of the Society was held Saturday morning, October 14, at 9:30 A.M. President Gibbs presided. Formal reports of the Secretary, Editor-of-Publications and Treasurer for the calendar year 1938* were laid before the Society. The reports were supplemented by in-

* Published in J. Opt. Soc. Am. **29**, 248–253 (1939).

formal reports of the changes since January 1, 1939. The reports were accepted.

President Gibbs discussed briefly plans for a new series of Progress Reports on various phases of optics, similar in plan to those published in early numbers of the Journal.

The Secretary presented the report of the Tellers for the 1939 election of officers as reported in the Minutes of the Board of Directors.

The Chairman of the Committee on Meetings and Programs was instructed to write to the speakers on the Symposium of the Optics of Vision expressing the thanks of the Society for the excellent papers presented.

The Secretary was instructed to write to the Lake Placid Club expressing the appreciation of the Society for its hospitality which had contributed to a highly successful and enjoyable meeting.

Contributed Papers

Sessions for the reading and discussion of contributed papers were held Thursday, October 12, at 9:30 A.M. and Saturday, October 14, at 10:00 A.M. The authors' abstracts of these papers are appended.

Registration

Members 55, Guests 39, Total 94.

Meeting of the Board of Directors

A meeting of the Board of Directors was held on Thursday, October 12 and Friday, October 13, 1939. Minutes of the meeting are appended.

L. B. Tuckerman, *Secretary*

Minutes of the Fourteenth Meeting of the Directors of the Optical Society of America, Incorporated

The Lake Placid Club, New York, October 12, 1939

BESIDES the transaction of routine business the following items of general interest to members of the Society were taken up.

Announcement by Secretary

K. S. Gibson appointed member of the Division of Physical Sciences of the National Research Council for a term to expire June 30, 1942.

Results of Letter Ballots

1. Award of Frederic Ives Medal for 1939 to A. H. Pfund.
2. Appointment of Dorothy Nickerson, H. M. O'Bryan and W. B. Van Arsdel as Tellers and A. Q. Tool as alternate to count the ballots for the 1939 election of officers and directors.

Election of Officers

The count of the official tellers showed that the following officers and directors had been elected:

For a term of two years beginning October 14, 1939
President K. S. Gibson
Vice President A. G. Worthing
For a term of four years beginning October 14, 1939
Members-at-large of the Board of Directors: J. W. Beams, Rudolf Kingslake

Membership

The Secretary reported that since the last meeting of the Directors two members had died, 23 new members had been elected as of January 1, 1939, and 2 as of January 1, 1940.

Transfer from Associate to Regular Membership

The following were transferred from associate to regular membership:

Eugene Wm. Beggs	(1164 R)
Janet Howell Clark	(337 R)
Robert C. Colwell	(525 R)
Alan A. Cook	(957 R)
Ralph K. Day	(1139 R)
Alexis A. B. Dember	(1160 R)
Ralph M. Evans	(563 R)
Glenn Ansel Fry	(1184 R)
Carl W. Gartlein	(1154 R)
Hans Goldmann	(1147 R)
Walter C. Granville	(944 R)
Ernest H. Greppin	(1169 R)
V. G. W. Harrison	(1149 R)
H. Orlo Hoadley	(1007 R)
Andrew Hustrulid	(1024 R)
Oran E. Miller	(1017 R)
John F. Morrow, Jr.	(1034 R)
Bradford Noyes, Jr.	(459 R)
Harold Osterberg	(1183 R)
F. W. Reynolds	(465 R)
Shirleigh Silverman	(1171 R)
Luville T. Steadman	(958 R)
Arthur F. Turner	(1106 R)
Jack H. Waggoner	(1090 R)
Charles M. Wilson	(1165 R)
Harvey A. Zinszer	(1141 R)

Financial Status of the Society

The Treasurer gave an informal report which showed that the financial condition of the Society continued satisfactory.

Appointment of Associate Editors

The following were appointed as Associate Editors of the Journal for terms of six years from January 1, 1940 to December 31, 1946.

K. S. Gibson	E. D. Tillyer
P. E. Klopsteg	H. T. Wensel
W. B. Rayton	F. E. Wright

Organization of Student Branch

The Chairman of the Committee on Student Memberships reported that the organization of the student branch at the University of Rochester was nearly completed.

Continuation of additional support to the American Institute of Physics

It was voted that:

"Contingent upon the other Member Societies continuing for another year their added support to the American Institute of Physics, the Treasurer is authorized to pay to the Institute for the year 1940 a service charge of twenty (20%) percent instead of the charge of fifteen (15%) percent provided for in the contract signed December 18, 1933, as amended in 1934."

Adolph Lomb Medal

The Directors decided that the Society should establish an Adolph Lomb Medal to be awarded not oftener than once every two years to a person under the age of thirty years who shall have completed some noteworthy accomplishment in the field of optics.

It was voted that:

"A committee be appointed to formulate the conditions of the award of an Adolph Lomb Medal, to report at the February, 1940, meeting of the Board of Directors.

"A committee be appointed to supervise the preparation of the design of an Adolph Lomb Medal and to report their recommendations at the February, 1940, meeting of the Board of Directors.

"A committee be appointed to recommend a candidate to receive the Adolph Lomb Medal in October, 1940, this committee to report progress at the February, 1940, meeting of the Board of Directors."

Twenty-Fifth Annual Meeting

The Directors formally approved the holding of the Twenty-Fifth Annual Meeting during October, 1940, in Rochester, New York, where the Society was founded in 1916.

Committee on Placement Service of the American Institute of Physics

R. C. Gibbs reported that he had been appointed Chairman of a committee to advise the American Institute of Physics in the improvement of its placement service.

L. B. Tuckerman, *Secretary*

Program of Sessions for Contributed Papers

(TITLES AND ABSTRACTS OF PAPERS)

1. Apparatus for Investigation of the Aurora Borealis. C. W. GARTLEIN, *Cornell University.* (15 minutes.)

The National Geographic Society-Cornell University Study of the Aurora 1938–1941, has completed the first year of its program. This is a comprehensive study of the aurora to determine heights, geographic location, spectra and intensity of auroras near Ithaca as well as distribution of the auroras in time and over the United States and Canada. Most of the apparatus has been in use for some time and has given complete satisfaction.

The four height measuring stations, two at Ithaca, New York, one at Geneva (Hobart College) and Hamilton, New York (Colgate University) take simultaneous photographs, two stations at a time, on 9×12-cm plates. Each plate contains six photographs. Plates used are Eastman Spectroscopic 1-C special. Eastman Tri-X Pan film, though not yet used, appears better according to sensitometer tests.

The Cornell-Colgate baseline is equipped with special cameras which use Ektar lenses *f:* 2 aperture and 45-mm focus. The Cornell-Hobart baseline uses Norwegian auroral cameras equipped with Astro-lenses *f:* 1.25, 50-mm focus. Over 1000 successful photographs have been taken among which about 200 pairs are suitable for height measurement. The minimum exposure made is $\frac{1}{2}$ second.

Color photographs have been made with a Leica Camera *f:* 1.5 lens on Kodachrome. Moderately satisfactory color photographs of red rays, red surfaces and colored bands have been obtained.

A motion picture camera with *f:* 1.5 lens has been adapted to take exposures of approximately 30 sec. duration at 30-sec. intervals to show the development of bands, drift of rays and similar studies.

Two spectrographs have been used. A precision spectrograph, whose resolving power is limited by the grain of the fast plates, consists of a 380-mm focus collimator, two 60° prisms of 10-cm faces, and a camera lens of 10-cm focus aperture *f:* 1.65. The visible spectrum is 14 mm in length.

A smaller spectrograph with collimator, one prism and a camera lens of 50-mm focus *f:* 1.65 aperture is arranged so the plate can be moved continuously through the focal plane giving an exposure of 40 minutes to each part of the record. This provides a record of spectral changes with time.

A simple exposure meter specially built for these low brightnesses, is used to control exposures.

A photoelectric recording photometer to measure auroral light has been under construction for some time. It has been tried once and had much greater sensitivity than necessary though the stability must and can be improved. It consists of a sector disk interrupter, photo-cell with six stage multiplier, a. c. amplifier, synchronous commutator rectifier and graphical recorder.

This report will be illustrated by photographs of the apparatus and of typical auroral forms and spectra.

2. Approximate Spectral Energy Distribution of Skylight. KASSON S. GIBSON, *National Bureau of Standards.* (10 minutes.)

The spectral energy distribution of skylight of a color temperature approximating 7400°K was recently desired.[1] Since blue sky is produced by scattering of the sun's radiant energy, probably in accord with the inverse λ^4 relation (Rayleigh scattering),[2] and since the distribution of the energy emitted from the completely overcast sky, and probably also of that reflected from white clouds, is close to that of the sun outside the earth's atmosphere, judging from the respective colors, it seems reasonable to suppose that the spectral energy distribution of various phases of skylight can be approximated by combining the spectral energy distribution of the sun outside the atmosphere with that given by the $1/\lambda^4$ relation.

An accurate formulation of such a relation would include one or more functions based on the exponential relation of transmission to thickness. However, for additive mixtures of light from blue sky and white clouds the following simple relations should give good approximations to the true energy distributions. The approximations are probably less good for hazy or lightly overcast blue sky.

If S_0 be the relative spectral energy distribution for sun outside the atmosphere and S_b that of the bluest possible skylight, the assumed spectral distribution of this skylight is taken as

$$S_b = (C/\lambda^4)S_0.$$

If D be the relative spectral energy distribution of any phase of daylight bluer than S_0, it is further assumed that

$$D = k_1 S_0 + k_2 (C/\lambda^4) S_0,$$

where $k_1 + k_2 = 1.0$.

With values of S_0 based on Abbot's 1923 data,[3] values of D have been computed for k_2 equal to 0.0, 0.1, 0.2, . . . 1.0. Values of the trichromatic coefficients computed from these values of D lie on a straight line very close to the Planckian locus, as will be illustrated, with color temperatures[4] increasing from approximately 6200°K ($k_2 = 0.0$) to infinity ($k_2 = 0.9$ and 1.0). Interpolation enables the approximate spectral energy distribution for any measured color temperature of the sky to be derived.

[1] Dorothy Nickerson, "Artificial Daylighting Studies," Paper presented at Thirty-Third Annual Convention, I. E. S., August, 1939.
[2] R. W. Wood, *Physical Optics*, third edition (Macmillan, 1934), pp. 423–427.
[3] A. C. Hardy, *Handbook of Colorimetry* (Technology Press, 1936), Table 6, p. 22.
[4] D. B. Judd, "Estimation of Chromaticity Differences and Nearest Color Temperature on the Standard 1931 I. C. I. Colorimetric System," J. Opt. Soc. Am. **26**, 421 (1936).

3. The Spectral Distribution of Energy in Daylight. A. H. TAYLOR AND G. P. KERR, *Lighting Research Laboratory, General Electric Co.* (20 minutes.)

One of the authors has previously reported the results of a study of the color of daylight[1] at various times throughout a year, which showed that the color temperature of

different phases of daylight varies over a range from about 5000 to 25,000 degrees K. However, published data regarding the *spectral distribution* of energy for various phases of daylight are very meager. Kimball[2] and Gage[3] have published some data but they are far from complete.

By means of a Hilger spectrometer, a photo-tube and amplifier the authors have made measurements of energy distribution for a wide variety of daylight conditions. These will be presented in the form of graphs, with computed color temperatures. Such data are of considerable importance in the use of natural daylight and the production and use of artificial daylight.

* The present paper will appear in full in J. Opt. Soc. Am.
[1] Trans. I. E. S. **25**, 154 (1930).
[2] Proceedings International Congress on Illumination 501 (1928).
[3] Trans. I. E. S. **34**, 316 (1939).

4. The Use of Ulbricht Sphere Theory in the Computation of Illumination. FRANK BENFORD, *General Electric Company.* (20 minutes.)

The computation of illumination received from extended sources involves some very complicated equations and for this reason this type of computation is seldom made unless a suitable chart or nomograph is available. If the theory of the Ulbricht sphere is assumed, it is easily shown that a plane surface may be derived with all the illumination properties of the sphere and thus arrive at a workable chart for many difficult illumination problems.

Such a chart has the further advantage of making it possible to take in account many details such as external obstructions and variations of sky brightness with angular elevation, and this permits a closer analysis and study than is possible by either equations or monograms.

The theory of the Ulbright sphere applies directly to the brightness of images on the retina, and several other applications of an engineering nature are here presented.

5. Sources of Error in Operation of the Multipurpose Reflectometer.* RICHARD S. HUNTER, *National Bureau of Standards.* (15 minutes.)

The multipurpose reflectometer, its plan and its various applications have been described in three papers presented before the Society.[1] Use of this instrument has revealed sources of error which should be of interest, not only to users of the device, but also to those using other pieces of apparatus employing barrier-layer photo-cells. Twelve sources of error are evidenced by inconstant operation and failure of the instrument to duplicate its readings, and six produce inaccuracies which cannot be detected without measurements of the same quantities on other instruments.

I. Sources of inconstancy:
  A. From photo-cells
    1. Change of photo-cell current with temperature
    2. Change of photo-cell current with time of exposure to light (fatigue)
    3. Change of relative photo-cell current with change of intensity of source (failure of pairings)
    4. Failure of contacts within photo-cells
  B. From the design of the instrument
    5. Presence of stray light
    6. Lack of structural rigidity
    7. Presence of dust
    8. Spectral change of source with line voltage (important only with highly selective samples)
  C. From characteristics of sample
    9. Non-uniformity of reflecting surface
    10. Curvature of reflecting surface
    11. Variation of apparent reflectance with orientation in plane of reflecting surface
    12. Impermanence
II. Other sources of error:
    13. Inaccurate instrument scales
    14. Inaccurately calibrated standards
    15. Inexact duplication of specified geometric conditions of measurement
    16. Loss of light in translucent sample
    17. Failure of spectral responses of source-filter-photo-cell combinations to duplicate those required
    18. Fluorescence of sample

The multipurpose reflectometer was designed so as to hold these errors to a minimum, but certain precautions are required for its successful operation. The magnitude of the errors possible through neglect of these precautions will be illustrated by specific cases.

[1] R. S. Hunter, J. Opt. Soc. Am. **27**, 225 (1937); **28**, 51 and 179 (1938).
* The present paper will appear in full in Nat. Bur. Stand. J. Research.

6. Some Problems and Methods of Dyestuffs Automatic Spectrophotometry.* I. H. GODLOVE, *E. I. du Pont de Nemours & Co.* (20 minutes.)

No general application of automatic spectrophotometry to dyestuff control has been reported in the literature. We have been applying the Hardy-G.E. photoelectric instrument to routine standardization as well as to research problems. Difficulties are encountered for several reasons; e.g.: (1) thousands of pounds of each dyestuff may be held in mixers between the (preliminary) morning and (adjusted) afternoon test, tyeing up equipment, hence speed is essential; (2) speed is also essential because of the large daily number of tests required; (3) the precision of rapid, convenient methods of making up solutions is not as great as the precision of the instrument; (4) for several reasons, which will be discussed, the relative "strengths" and chromaticities of individual lots and standards frequently do not agree with those obtained by dyeing tests, and in such cases a measure of "true dye" content is not necessarily what is wanted; (5) comparison of spectral curves for dye strength is most conveniently made with the logarithmic cam, but for chromaticity with the normal cam; (6) difficulties arise from the necessity of using hot solutions, acid or alkaline buffered and extremely dilute, conditions often not obtained in practical use; (7) a very large fraction of the thousand-odd dyes of a commercial line are insoluble in water or other ordinary solvents. Other difficulties and typical results will be illustrated.

On the side of research and special applications, several techniques have been investigated. A promising one, which combines spectrophotometry with selective adsorption of dyes of a mixture on finely-divided white substances, will be illustrated with lantern slides. Important applications of the method have been made.

[1] Draves, J. Opt. Soc. Am. **21**, 336–46 (1931).
[2] Hardy, J. Opt. Soc. Am. **25**, 305–11 (1935).
[3] Michaelson, J. Opt. Soc. Am. **28**, 365–71 (1938).
* The present paper will appear in full in J. Opt. Soc. Am.

7. A Duboscq Colorimeter Which Reads Results Directly. ROGER S. ESTEY, *Spencer Lens Company.* (10 minutes.)

Although the essential design of the Duboscq colorimeter has remained unchanged for years, many refinements of detail but few major improvements have been added. In an effort to make a fundamental advance in the development of this instrument the basic design and subsequent changes have been critically reviewed. From this has emerged a colorimeter which successfully breaks away from tradition in many respects. The illumination in this new instrument is provided by an electric lamp in the base and for simplicity no arrangements are made for the use of any other illuminant. The optical system utilizes oblique upper surfaces on the plungers which by refraction direct the beams into a biprism. This construction makes possible a forward tilt to the eyepiece without requiring additional optical elements. The millimeter scales for measuring cup depth are abandoned in favor of arbitrary equal part scales which extend from 0 to 14.00 units. Similar scales and drums are used to measure the depth of each cup. The units and fractions read on separate drums. The fractional drums are graduated to 0.05 units and 0.01 units can be read by estimation. This choice of scale interval places 10.00 near the center of the upper half of the range and permits the result to be determined over the range of 0 to 140 percent without calculation. The cups have a useful depth of about 55 mm. In use, the depth of the sample solution is set at 10.00 and the color match obtained by varying the depth of the standard solution. The result is evident by inspection of the scale drum on the standard side. For use with glass standards or with occasional concentrations for which a depth of 10.00 units is unsuited, other depths may be used, both scales read, and the result obtained from the customary calculation. Other features contributing to the accuracy and convenience of the instrument will be demonstrated.

8. An Analysis of the Original Munsell Color System. JOHN E. TYLER AND ARTHUR C. HARDY, *Massachusetts Institute of Technology.* (15 minutes.)

Glenn and Killian have made an analysis (unpublished) of the Munsell Color System as it existed in 1934. A similar analysis has now been made of the Munsell System as it existed at the time of Professor Munsell's death, this analysis being based upon spectrophotometric data obtained in 1919 at the National Bureau of Standards. Because the original Munsell System was constructed by the use of Maxwell disks, the *hue* of that system is identical with dominant wave-length. By definition, *value* in the original Munsell System is the square root of the photometric brightness expressed in percent. There is also a simple mathematical relationship between *chroma* in the original Munsell System and excitation purity. In this paper the original colors of the Munsell System are represented on the familiar I. C. I. chromaticity diagram and are compared with the results of Glenn and Killian.

9. The Effect of Distortion in Aerial Mapping by the Multiple Projection Method.* H. G. OTT, *Bausch & Lomb Optical Co.* (15 minutes.)

The distortion of the optical systems used in the method of aerial mapping consisting of multiple projection of overlapping images of aerial photographs tends to warp the resulting plastic model out of its true shape. The relations between the distortion and the warped shape of the model are developed. It is shown that after the distortion is reduced to a minimum, the balance of residual distortion has a significant effect on the warp of the model. The best type of balance of residual distortion is indicated.

* The present paper will appear in full in J. Opt. Soc. Am.

Invited Papers on Optics of Vision

10. Comparative Anatomical Studies of the Eye with Especial Reference to the Photoreceptors. DR. S. R. DETWILER, *College of Physicians and Surgeons, Columbia University.*

11. The Nerve Messages in the Fibers of the Visual Pathway. DR. H. K. HARTLINE, *School of Medicine, University of Pennsylvania.*

12. Chemistry of Visual Systems. DR. GEORGE WALD, *Biological Laboratories, Harvard University.*

13. Recent Researches in Color Vision. DR. BRIAN O'BRIEN, *University of Rochester.*

14. Aniseikonia. DR. GORDON H. GLIDDEN AND DR. KENNETH N. OGLE, *Dartmouth Medical School.*

15. Luminosity Methods and Results. DR. KASSON S. GIBSON, *National Bureau of Standards.*

16. The Supra-Threshold Realm of Seeing. DR. MATTHEW LUCKIESH AND DR. FRANK K. MOSS, *Lighting Research Laboratory, Nela Park.*

17. Total Hemispherical Emissivities by a Parallel Plate Method. A. G. WORTHING, *University of Pittsburgh.* (15 minutes.)

This method is applicable only to opaque solids whose surfaces can be maintained by thermal conduction at known temperatures without appreciable temperature variation in the region through which the heat is being conducted. Painted or coated metallic surfaces are particularly suitable for the method.

Between two such plates slightly separated and with the space between evacuated, there is exchange of energy by radiation in accord with

$$1/\epsilon_{t1}+1/\epsilon_{t2}=1+\frac{\sigma(T_1^4-T_2^4)}{W}$$

where ϵ_{t1} and ϵ_{t2} are the total hemispherical emissivities of the two bodies, T_1 and T_2, their temperatures, and W the rate of transfer of energy per unit area from one to the other. This equation assumes that the changes in ϵ_{t1} and ϵ_{t2} in the temperature range T_1 and T_2 are negligible. If the two surfaces are identical there follows the simplification that

$$1/\epsilon_t=\tfrac{1}{2}+\frac{\sigma(T_1^4-T_2^4)}{2W}.$$

In practice the surface of one plate may be held at the constant temperature T_1 while the temperature of the other plate, initially below that of the surroundings T_2, increases gradually and passes through T_2. Observation of the rate dT/dt at temperature T_2 combined with the mass and the specific heat of the second plate yields W at once. There follows by means of the formulas given, a method of evaluating ϵ_{t2} if ϵ_{t1} is known or ϵ_t if both plates have the same surface.

Though the method is closely allied to Shakespeare's procedure in determining the fourth power radiation constant, its possibility as a means of determining total radiation emissivities seems to have been overlooked.

18. A Hydrogen Arc for Absorption Spectroscopy.* A. J. ALLEN AND R. G. FRANKLIN, *Biochemical Research Foundation of The Franklin Institute.* (10 minutes.)

Hydrogen discharge tubes giving a fairly continuous source of ultraviolet radiation have been found to be satisfactory for ultraviolet absorption spectroscopy. The low voltage arc to be described is a modification of the arc discribed by Duffendack and Manley[1] and Munch[2] and has proved to be exceedingly reliable and convenient. It is simple to operate and its intensity is comparable to that of a standard iron arc. It is suitable for many experiments requiring a continuous spectrum in the ultraviolet region and its cost of construction is low.

* The present paper will appear in full in J. Opt. Soc. Am.

[1] O. S. Duffendack and J. J. Manley, J. Opt. Soc. Am. **24**, 222 (1934).

[2] R. H. Munch, J. Am. Chem. Soc. **57**, 1863 (1935).

19. A Thallous Sulphide Photo-E.M.F. Cell.* FOSTER C. NIX AND ARNOLD W. TREPTOW, *Bell Telephone Laboratories.* (Introduced by E. F. Kingsbury). (20 minutes.)

A photo-e.m.f. effect is observed in Tl_2S produced by partial sulphurization of a thallium disk or of an evaporated thallium layer. The photosensitivity is observed to be as high as $6000\cdot10^{-6}$ ampere/lumen. Both open-circuit-voltage and short-circuit current are observed in some cells to be directly proportional to the incident light intensity at all temperatures. In other cells, especially those displaying high photo-response, the open-circuit-voltage and short-circuit current tend to saturate at high light intensities. The spectral photosensitivity curve possesses a maximum at 9800A±200A with long and short wave thresholds at 15,000 and 6500A, respectively. The short-circuit current *increases* with *decreasing* temperature down about −45°C and then decreases continuously down to −180°C. The position of the maximum varies somewhat from cell to cell. The photoelectric output of the cells when measured as a function of frequency of interruption of the incident light displays a recession of 75 percent at 20,000 cycles.

* The present paper will appear in full in J. Opt. Soc. Am.

20. The Triboluminescence of Sugar and of Ice.* FRANCES G. WICK, *Vassar College.* (10 minutes.)

Necco wafers, Life Savers and other crystalline sugar candies, when broken or ground in a mortar, give off light which, according to Harvey,[1] varies in intensity with the flavor, wintergreen being most intense. Harvey suggests that this is because wintergreen oil is luminescent when excited by light from a spark and that sparks take place when sugar crystals are broken with the emission of triboluminescence. The sparks excite fluorescence in the wintergreen.

This paper describes observations of the effect of an iron spark in exciting luminescence in oils such as are used in flavoring and also in sugar crystals made from solutions to which such oils have been added. Sugar crystals made with the addition of different luminescent DuPont dyes were also observed. Experiments were made at ordinary and low temperatures down to that of liquid air.

The effect of the spark discharge from iron terminals upon pure sugar crystals including cane sugar, maltose, and lactose is to produce dim phosphorescence at room temperature and much stronger phosphorescent at liquid air temperature with differences in intensity for the different kinds of sugar. The intensity of triboluminescence of sugar is greater at low temperatures than at ordinary temperatures and the addition of oils or dyes which are fluorescent under the spark intensifies the triboluminescence. Oils such as wintergreen, bergamont, lemon, lavendar and peppermint were used.

Crystals made from cane sugar, cubes of ordinary granulated sugar, and crystalline candies such as Necco wafers and Life Savers, when dropped into liquid air, emit light which is evidently caused by the cracking of the crystals with the sudden change in temperature. When oil of wintergreen is placed on the surface of the crystal or wafer before it is dropped into the liquid air a flickering light is emitted from the wintergreen as the specimen cools making the total emission much stronger. It looks as if sparks are emitted when the crystals crack and that these sparks excite fluorescence in the wintergreen. Other

luminescent oils show a similar effect but wintergreen is strongest because its fluorescence under the spark is most intense.

Ice is phosphorescent under the iron spark and it is also triboluminescent at a temperature a little above that of liquid air. The intensity of the triboluminescence is increased by exposing the ice to the iron spark at a low temperature before grinding it in a mortar.

These experiments confirm the explanations of Harvey[1] and the assumptions made by Wick[2] as to the different factors involved in the emission of triboluminescence.

This work was done at Cornell University with assistance from a Grant from the Rumford Fund of the American Academy of Arts and Sciences.

* The present paper will appear in full in *J. Opt. Soc. Am.*
[1] Harvey, Science **90**, 35 (1939).
[2] Wick, J. Opt. Soc. Am. **27**, 284 (1937).

21. The Optical Properties of Semitransparent Sputtered Films Determined by Interference of Light.* J. B. NATHANSON AND C. L. BARTBERGER, *Carnegie Institute of Technology.* (15 minutes.)

A study has been made of the optical properties of semitransparent films of variable thickness produced by sputtering in hydrogen. The masses of the films have been computed from the assumed optical constants and densities of the metals, and from the locations of the interference fringes, when the films are viewed by reflected monochromatic light at approximately normal incidence. With few exceptions the computed masses are much greater than the masses determined directly by weighing. The differences between the computed and weighed masses can be accounted for by assuming higher refractive indices and lower densities of the films than those assumed for the bulk metals. Such would be the case if the films were composed of compounds, e.g., oxides of the metals. This has been verified by a colorimetric analysis of the iron films. Further confirmation is supplied by a study of the optical dispersion of the films, which seems to be characteristic of transparent insulators rather than of the metals. High refractive indices are also called for from a consideration of the aging of a platinum film. A mathematical analysis of the reflectivity of the films has been made, based upon the consideration of multiple reflections within the films. To account for the number of fringes observed, very low indices of absorption must be assumed, such as possessed by transparent insulators.

* The present paper will appear in full in October issue of J. Opt. Soc. Am.
[1] J. B. Nathanson, J. Opt. Soc. Am. **23**, 388 (1933).
[2] K. Prosad and B. N. Ghosh, Ind. J. Phys. **10**, 49 (1936).

22. Hyperfine Structure Deviations in Sb^{121} and Sb^{123}.* D. H. TOMBOULIAN AND R. F. BACHER, *Cornell University.* (Introduced by R. C. Gibbs.) (15 minutes.)

Measurements of the hyperfine structure of the line $\lambda 5895 (5s^2 5p6p^3D_1 \rightarrow 5s^2 5p6s^3P_0)$ in SbII, using a Fabry-Perot interferometer, have shown that there are deviations from the interval rule. By using 2.5, 3.5, and 4.0-mm separators, the hyperfine structure intervals of the $^3D_1$ level of $Sb^{121}(I=5/2)$ are found to be 0.680 and 0.483 cm^{-1}. The corresponding intervals in $Sb^{123}(I=7/2)$ are 0.481 and 0.360 cm^{-1}. The error in these measurements is of the order of 0.001 cm^{-1}. The ratio of the two separations gives 1.41 and 1.34 for the light and heavy isotope, respectively, whereas the corresponding ratios on the basis of the interval rule are 1.400 and 1.286. The deviation for the light isotope is within the experimental error, but the deviation for the heavy isotope can be considered as definite evidence that the interval rule is not obeyed. The observed deviation cannot be accounted for entirely on the basis of perturbing effects of neighboring levels but must be ascribed to the presence of an electric quadrupole moment of the nucleus.

* The present paper will appear in full in Phys. Rev.

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